



**Achieve ADP Algebra II  
End-of-Course Exam Content Standards  
with Comments & Examples**

---

Core and Optional Modules

January 2010

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010 Change Log

In order to help track where changes and additions have been made to wording and examples throughout the document, this change log has been included.

| Section           | Column | Description  | Date         |
|-------------------|--------|--|--------------|
| Table of Contents | N/A    | Updated.   | January 2010 |
| Benchmark O2a     | Right  | Example fixed.   | January 2010 |
| Benchmark O3f     | Right  | Example added.   | January 2010 |
| Benchmark P2a     | Right  | First bullet clarified with statement, "For a polynomial function that results from transformations on a power function, that particular power function would be considered the parent function of that polynomial, such as $y = x^3$ is the parent function of $y = 3(x - 2)^3 - 1$ ." Example changed. | January 2010 |
| Benchmark P2e     | Right  | Example added.   | January 2010 |
| Benchmark P2f     | Right  | Bullet added, "Solve problems involving direct, inverse, and joint variations."  | January 2010 |
| Benchmark F1a     | Right  | Example changed.   | January 2010 |
| Change Log        | N/A    | Created.   | October 2008 |
| Table of Contents | N/A    | Created.   | October 2008 |
| Background        | N/A    | Front matter updated with current information.   | October 2008 |
| Benchmark O1b     | Right  | Second example added. Bullet from Benchmark O2b adjusted, "Divide irrational numbers, involving radicals, using conjugates, such as $a + \sqrt{b}$ and $a - \sqrt{b}$ ."   | October 2008 |
| Benchmark O2b     | Right  | Second and third bullets combined and adjusted to fit benchmark, "Divide complex numbers using conjugates, $a + bi$ and $a - bi$ ."  | October 2008 |
| Benchmark O3a     | Right  | Altered first and second example.  | October 2008 |
| Benchmark O3c     | Right  | Altered first example.   | October 2008 |
| Benchmark P1c     | Left   | Benchmark adjusted to "Describe <i>and represent</i> ..."  | October 2008 |
| Benchmark P2b     | Right  | Corrected general form of polynomial.  | October 2008 |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|                                    |       |   |              |
|------------------------------------|-------|---|--------------|
| Benchmark X1b                      | Left  | Corrected benchmark language, "Represent exponential <del>expressions</del> <i>equations</i> in multiple forms."  | October 2008 |
| Benchmark X1c                      | Left  | Benchmark adjusted to "Describe <i>and represent</i> the effect that changes in <i>the</i> parameters of"   | October 2008 |
| Benchmarks I1d and I1e             | Left  | Corrected notation in formulas.   | October 2008 |
| Graphing Benchmarks                | Right | Clarification added to indicate what is expected of student responses: "For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc)." | October 2008 |
| Benchmarks O3d, O3e, E1b, C1c      | Right | Bullets that were written for limitations of assessment, not instruction were written moved to the bottom of the box and labeled as "Assessment Limitations."   | October 2008 |
| Various Benchmarks                 | Right | Some statements and examples on the right hand side were reworded for clarity of expectations.  | October 2008 |
| Notation Information               | N/A   | Updated and added to document.  | October 2008 |
| Expectations of Geometry Knowledge | N/A   | Added.  | October 2008 |
| Calculator Policy                  | N/A   | Removed.  | October 2008 |
|                                    |       |   |              |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010  
Table of Contents**

|  |    |
|--|----|
| Change Log _____   | 2  |
| Table of Contents _____  | 4  |
| Overview of Content Standards _____  | 5  |
| About this document: _____   | 6  |
| Background: _____  | 6  |
| Core: Operations on Numbers and Expressions _____                          | 9  |
| Core: Equations and Inequalities _____                                     | 12 |
| Core: Polynomial and Rational Functions _____                              | 17 |
| Core: Exponential Functions _____  | 21 |
| Core: Function Operations and Inverses _____                               | 23 |
| Module: Data and Statistics _____  | 26 |
| Module: Probability _____  | 29 |
| Module: Logarithmic Functions _____  | 33 |
| Module: Trigonometric Functions _____                                      | 35 |
| Module: Matrices _____   | 37 |
| Module: Conic Sections _____   | 39 |
| Module: Sequences and Series _____   | 41 |
| ADP Algebra I and Algebra II End-of-Course Exam Notation Information _____ | 43 |
| ADP Algebra II End-of-Course Exam Expectations of Geometry Knowledge _____ | 45 |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010  
Overview of Content Standards**

**Exam Core Content:**

**O: Operations on Numbers and Expressions**

- O1. Real numbers
- O2. Complex numbers
- O3. Algebraic expressions

**E: Equations and Inequalities**

- E1. Linear equations and inequalities
- E2. Nonlinear equations and inequalities

**P: Polynomial and Rational Functions**

- P1. Quadratic functions
- P2. Higher-order polynomial and rational functions

**X: Exponential Functions**

- X1. Exponential functions

**F: Function Operations and Inverses**

- F1. Function operations and composition
- F2. Inverse functions
- F3. Piecewise functions

**Optional Module Content:**

**S: Data and Statistics**

- S1. Summarization and comparison of data sets
- S2. Interpretation and communication through data

**R: Probability**

- R1. Permutations, combinations, and probability
- R2. Probability distributions

**L: Logarithmic Functions**

- L1. Logarithmic expressions and equations
- L2. Logarithmic functions

**T: Trigonometric Functions**

- T1. Trigonometric functions

**M: Matrices**

- M1. Matrix arithmetic
- M2. Solving systems of equations using matrices
- M3. Matrix transformations
- M4. Vectors

**C: Conic Sections**

- C1. Conic sections

**I: Sequences and Series**

- I1. Arithmetic and geometric sequences and series
- I2. Other types of iteration and recursion

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples

January 2010

## About this document:

*This version of the Algebra II End-of-Course Exam Content Standards includes two columns: The first column contains the standards, objectives and benchmarks; the second column includes explanatory comments and examples. These comments and examples are meant to add clarity to the meaning of the benchmarks for teachers and test item writers. Examples are provided only when necessary for clarity and are not meant to be exhaustive or to be used as sample test items. In some instances, in the standards and in the explanations, the word “including” is used followed by a list. The word “including” does not translate to “all inclusive” but rather means “including but not limited to.” Some of the benchmarks have “assessment limitations” which means that the stated content is not tested on the ADP Algebra II End-of-Course Exam. However, this does not imply that teachers should not teach or expand on this content.*

**Background:** The American Diploma Project (ADP) Network includes states dedicated to making sure every high school graduate is prepared for college and a career. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments, and curriculum, and better aligning these expectations with the demands of postsecondary education and careers.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam at the Algebra II level. These states were planning to require or strongly encourage students to take an Algebra II level course in order to better prepare them for college and careers, as Algebra II or its equivalent serves as a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using a common end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a high quality test faster and at lower cost to each state, and to compare their performance and progress with one another.

The ADP Algebra II End-of-Course Exam serves three main purposes:

1. **To improve curriculum and instruction**—and ensure consistency within and across states. The test will help classroom teachers focus on the most important concepts and skills in an Algebra II or equivalent class and identify areas where the curriculum needs to be strengthened. Teachers will get test results back within three weeks of administering the test, which will provide sufficient time to make the necessary adjustments for the next year’s course.
2. **To help colleges determine if students are ready to do credit-bearing work.** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to enter and succeed in first-year, credit-bearing math courses. Postsecondary institutions will be able to use the results of the test to tell high school students whether they are ready for college-level work, or if they have content and skill gaps that need to be filled before they enroll in college. This information should help high schools better prepare their students for college, and reduce the need for colleges to provide costly remediation courses.

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

3. **To compare performance and progress among the participating states.** Having agreed on the core content expectations for courses at the Algebra II level, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement, and evaluate the impact of state strategies for improving secondary math achievement.

The Algebra II End-of-Course Exam will consist of a common core, which will be taken by students across participating states. States not part of the development group may also decide to purchase and administer this test. In addition to the core algebraic content, seven optional modules will be available to states to enrich the core with content which may currently be in their Algebra II or equivalent course that is deemed important to colleges and the workplace. The content in the core and modules is described below.

**The Core Algebra II End-of-Course Exam:** The core Algebra II End-of-Course Exam covers a range of algebraic topics. Successful students will demonstrate conceptual understanding of the properties and operations of real and complex numbers. They will be able to make generalizations through the use of variables resulting in facility with algebraic expressions. They will solve single and systems of linear equations and inequalities and will be able to use them to represent contextual situations. Successful students also will be able to demonstrate facility with estimating and verifying solutions of various non-linear equations, making use of technology where appropriate to do so. Finally, students will demonstrate knowledge of functions and their properties – distinguishing among quadratic, higher-order polynomial, exponential, and piecewise-defined functions – and recognize and solve problems that can be modeled by these functions. They will be required to analyze these models, both symbolically and graphically, and to determine and effectively represent their solution(s). There are a variety of types of test items developed that will assess this content, including some that cut across the objectives in a standard and require students to make connections and, where appropriate, solve rich contextual problems.

There are three types of items on the Algebra II End-of-Course Exam, including multiple choice (worth 1 point each), short answer (worth 2 points each), and extended response (worth 4 points each). At least one-third of the student's score will be based on the combined scores of the short-answer and extended-response items. Although the test is untimed, it is designed to take approximately 180 minutes, comprised of two 90 minute sessions, one of which will allow calculator use. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

**Optional Algebra II End-of-Course Exam modules:** Optional modules in Data and Statistics, Probability, Logarithmic Functions, Trigonometric Functions, Matrices, Conic Sections, and Sequences and Series have been developed. One or more of these modules may be used by states, districts, or schools whose curriculum includes these topic areas in Algebra II or an equivalent course. Each module will include the same types of items as the core exam: multiple-choice, short answer, and extended response. Fifty percent of the student's score on the modules will be based on the combined scores of the short answer and extended response items. A student should be allowed at least 30 minutes for each module. Calculators are allowed on all of the modules.

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

**Algebra II End-of-Course Exam calculator policy:** The appropriate and effective use of technology is an essential practice in the Algebra II classroom. At the same time students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra II End-of-Course Exam and should be expected in classrooms where students are working at the Algebra II level. It is therefore important that the Algebra II End-of-Course Exam reflects both practices. For purposes of the Algebra II End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions and the use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should not use a calculator that is new or different for them on the Exam but rather should use the calculator they are accustomed to and use every day in their classroom work. For more information about technology use on the Algebra II End-of-Course Exam, see the ADP Algebra End-of-Course Exams Calculator Policy at [www.achieve.org/AssessmentCalcPolicy](http://www.achieve.org/AssessmentCalcPolicy).

It will be necessary to clear the calculator memory, including any stored programs and applications, on all calculators both before and after the exam. Please be advised that the clearing of the calculator memory may permanently delete stored programs or applications. Students should be told prior to the test day to store all data and software they wish to save on a computer or a calculator not being used for the test. In some states, an IEP or 504 Plan may specify a student's calculator use on this Exam. Please check with your state's Department of Education for specific policies or laws.

**Algebra II level curriculum:** Function modeling and problem solving is the heart of the curriculum at the Algebra II level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables, or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information, and decide whether an exact or approximate answer is appropriate, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

The standards included in this document are intended to reflect this curricular focus and to guide the work of the test designers and the test item developers. It is also the case that curriculum at the Algebra II level will include content and processes not included on the Algebra II End-of-Course Exam, as some are not easily assessed on an end-of-course exam of this nature. Problems that require extended time for solution should also be addressed in the Algebra II level classroom, even though they cannot be included in this end-of-course exam.

**Algebra II level classroom practices:** Effective communication using the language of mathematics is essential in a class engaged in Algebra II level content. Correct use of mathematical definitions, notation, terminology, syntax, and logic should be required in all work at the Algebra II level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in these Algebra II End-of-Course Exam content standards.

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Core: Operations on Numbers and Expressions  
Priority: 15%**

Successful students will be able to perform operations with rational, real, and complex numbers, using both numeric and algebraic expressions, including expressions involving exponents and roots. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples  |
|--|--|
| <b>O1. Real numbers</b>  |  |
| a. Convert between and among radical and exponential forms of numerical expressions.   | <ul style="list-style-type: none"> <li>Convert between expressions involving rational exponents and those involving roots and integral powers.<br/><i>Example:</i> <math>5^{\frac{3}{2}} = \sqrt{5^3} = 5\sqrt{5}</math> ; <math>\sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}</math>.</li> </ul>   |
| b. Simplify and perform operations on numerical expressions containing radicals.   | <ul style="list-style-type: none"> <li>Convert radicals to alternate forms and use the understanding of this conversion to perform calculations with numbers in radical form.<br/><i>Example:</i> <math>\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}</math></li> <li>Divide irrational numbers, involving radicals, using conjugates, such as <math>a + \sqrt{b}</math> and <math>a - \sqrt{b}</math>.<br/><i>Example:</i> <math>\sqrt[3]{1,024} + \sqrt[3]{16} + \sqrt[5]{4,096} = 8\sqrt[3]{2} + 2\sqrt[3]{2} + 4\sqrt[5]{4} = 10\sqrt[3]{2} + 4\sqrt[5]{4}</math></li> </ul>                                      |
| c. Apply the laws of exponents to numerical expressions with rational and negative exponents to order and rewrite them in alternative forms. | <ul style="list-style-type: none"> <li>Apply the properties of exponents in numerical expressions.<br/><i>Example:</i> <math>3^5 \cdot 3^2 = 3^{(5+2)} = 3^7 = 2187</math>, <math>\frac{3^5}{3^2} = 3^{(5-2)} = 3^3 = 27</math>,<br/><math>(3^5)^2 = 3^{(5 \cdot 2)} = 3^{10} = 59049</math>, <math>3^{\frac{5}{2}} = (\sqrt{3})^5 = \sqrt{3^5} = 9\sqrt{3}</math></li> <li>Convert between expressions involving negative exponents and those involving positive exponents.<br/><i>Example:</i> <math>3^{-2} = \frac{1}{3^2} = \frac{1}{9}</math> ; <math>\frac{2^{-3}}{7^{-1}} = \frac{7}{2^3} = \frac{7}{8}</math></li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**O2. Complex numbers**

|   |  |
|---|--|
| <p>a. Represent complex numbers in the form <math>a+bi</math>, where <math>a</math> and <math>b</math> are real; simplify powers of pure imaginary numbers.</p> | <ul style="list-style-type: none"> <li>• Every real number, <math>a</math>, is a complex number because it can be expressed as <math>a+0i</math>.</li> <li>• Represent the square root of a negative number in the form <math>bi</math>, where <math>b</math> is real; simplify powers of pure imaginary numbers.</li> </ul> <p>Example: <math>\sqrt{-8} = 2i\sqrt{2}</math></p> <p>Example: <math>\sqrt{-256} = 16i</math></p> <p>Example: <math>i^5 = i</math></p> |
| <p>b. Perform operations on the set of complex numbers.</p>   | <ul style="list-style-type: none"> <li>• Add, subtract, and multiply complex numbers using arithmetic rules.</li> <li>• Divide complex numbers using conjugates, <math>a+bi</math> and <math>a-bi</math>.</li> </ul> <p>Example: <math>\frac{(5+4i)}{3-2i} = \frac{(5+4i)}{(3-2i)} \cdot \frac{(3+2i)}{(3+2i)} = \frac{15+22i+8i^2}{9-4i^2} = \frac{7+22i}{13} = \frac{7}{13} + \frac{22}{13}i</math></p>  |

**O3. Algebraic expressions**

|   |   |
|---|---|
| <p>a. Convert between and among radical and exponential forms of algebraic expressions.</p> | <p>Example: <math>x^{\frac{5}{4}} = \sqrt[4]{x^5} = x\sqrt[4]{x}</math></p> <p>Example: <math>\sqrt{25a^6b^4} =  25 ^{\frac{1}{2}} a ^{\frac{6}{2}} b ^{\frac{4}{2}} =  5  a ^3 b ^2</math></p> |
| <p>b. Simplify and perform operations on radical algebraic expressions.</p>                 | <p>Example: <math>\sqrt{x^2+6x+9} = \sqrt{(x+3)^2} =  x+3 </math></p>   |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|  |  |
|--|--|
| <p>c. Apply the laws of exponents to algebraic expressions, including those involving rational and negative exponents, to order and rewrite them in alternative forms.</p> | <p><i>Example:</i> <math>5a^4 \cdot 3a^3 = 15a^{(4+3)} = 15a^7</math>, <math>\frac{a^4}{a^3} = a^{(4-3)} = a</math>, <math>(a^4)^3 = a^{(4 \cdot 3)} = a^{12}</math></p> <p><i>Example:</i> <math>y^{-2} = \frac{1}{y^2}</math>, <math>z^{\frac{2}{3}} = \sqrt[3]{z^2} = (\sqrt[3]{z})^2</math></p> <p><i>Example:</i> <math>\frac{r^{-3}}{t^{-2}} = \frac{t^2}{r^3}</math>, <math>\frac{a^{-4}}{b^3} = \frac{1}{a^4 b^3}</math>, <math>(a^3 b^5)^2 = a^6 b^{10}</math></p>  |
| <p>d. Perform operations on polynomial expressions.</p>  | <ul style="list-style-type: none"> <li>• Division may be performed using factoring.</li> </ul> <p><i>Assessment Limitations:</i> Multiplication should be limited to multiplication of, at most, a binomial by a trinomial. In division, the divisor should be limited to a linear or factorable quadratic polynomial.</p>   |
| <p>e. Perform operations on rational expressions, including complex fractions.</p>   | <p><i>Example:</i> <math>\frac{(a+b)}{\left(\frac{1}{a} + \frac{1}{b}\right)} \div ab = \frac{(a+b)}{\frac{(b+a)}{ab}} \div ab = (a+b) \cdot \frac{ab}{(b+a)} \div ab = ab \div ab = 1</math>.</p> <p><i>Assessment Limitations:</i> Expressions should be limited to linear and factorable quadratic denominators. Complex fractions should be limited to simple fractions in numerators and denominators.</p>  |
| <p>f. Identify or write equivalent algebraic expressions in one or more variables to extract information.</p>  | <p><i>Example:</i> The expression, <math>C + 0.07C</math>, represents the cost of an item plus sales tax, while <math>1.07C</math> is an equivalent expression that can be used to simplify calculations of the total cost.</p> <p><i>Example:</i> <math>x - \frac{y^2}{x}</math> can be rewritten as <math>\frac{x^2 - y^2}{x}</math></p> <p><i>Example:</i> The expression, <math>0.81x - 16.2</math>, represents the final amount paid for an item of original price <math>x</math>. If sales tax is 8%, then the equivalent expression <math>1.08(0.75x - 15)</math> shows that \$15 was taken off the price after a 25% discount was taken.</p> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

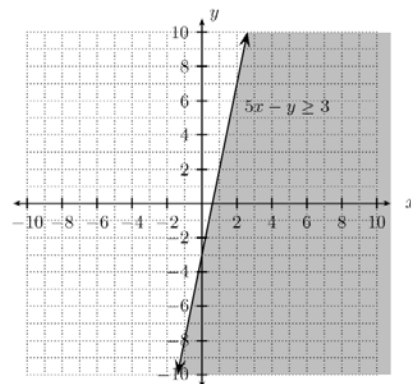
**Core: Equations and Inequalities**

**Priority: 20%**

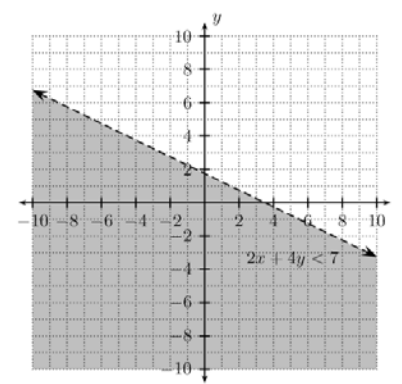
Successful students will be able to solve and graph the solution sets of equations and inequalities and systems of linear equations and inequalities. The types of equations are to include linear, linear absolute value, quadratic, exponential, rational, radical, and higher order polynomials; the types of inequalities are to include linear and quadratic. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. (Contextual test items will be limited to inequalities, systems of equations and inequalities, and those equations that do not represent a function.)

| Content Benchmarks  | Explanatory Comments and Examples  |
|---|--|
| <b>E1. Linear equations and inequalities</b>  |  |
| a. Solve equations and inequalities involving the absolute value of a linear expression.                    | <p><i>Example:</i> <math> x-6  \leq 8 \Leftrightarrow (x-6) \leq 8 \text{ and } -(x-6) \leq 8</math><br/> <math>\Leftrightarrow x \leq 14 \text{ and } x \geq -2 \Leftrightarrow -2 \leq x \leq 14</math></p>  |
| b. Express and solve systems of linear equations in three variables with and without the use of technology. | <p><i>Example:</i> Solve the system<br/> <math>2x + z = 11 \quad x - y + z = 6</math><br/> <math>x + 2y = 7 \quad \text{OR} \quad 2x + 2y - z = 1</math><br/> <math>2x - 4y + z = 3 \quad 3x + 4y + 3z = 4</math></p> <p><i>Assessment Limitation:</i> Systems in three variables should be limited to those with integer solutions and small integral coefficients.</p>   |
| c. Solve systems of linear inequalities in two variables and graph the solution set.                        | <p><i>Example:</i> Graph the solution set of the following system.<br/> <math>3x - 2y &lt; 7, \quad x \geq 0, \quad y \leq 0</math></p> <ul style="list-style-type: none"> <li>Recognize that the graphic solution of a linear inequality is either an open or closed half plane.</li> </ul> <p><i>Example:</i> Graphs (a) and (b) illustrate that the graph of a linear inequality is a half plane. Graph (c) illustrates a solution to the question: What is the set of points <math>(x, y)</math> that satisfies both <math>5x - y \geq 3</math> and <math>2x + 4y &lt; 7</math>?</p> |

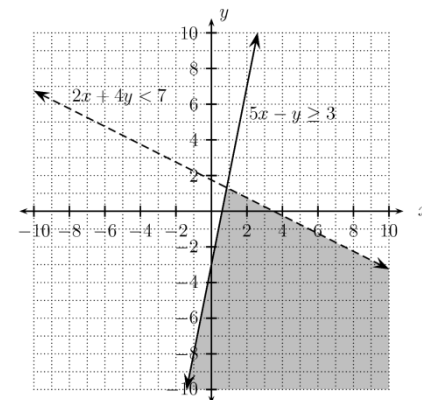
**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**



(a)



(b)



(c)

- For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

d. Recognize and solve problems that can be represented by single variable linear equations or inequalities or systems of linear equations or inequalities involving two or more variables. Interpret the solution(s) in terms of the context of the problem.

- This includes using and interpreting appropriate units of measurement and precision for the given application.
- Common problems are those that involve time/rate/distance, percentage increase or decrease, ratio and proportion, mixture problems and break-even problems.

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|   |   |
|---|---|
|   | <ul style="list-style-type: none"> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |
| <b>E2. Nonlinear equations and inequalities</b>   |   |
| <p>a. Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.</p> | <ul style="list-style-type: none"> <li>Use information gathered from a polynomial equation to determine the number and nature of its solutions: <ul style="list-style-type: none"> <li>The number of roots, real and complex and counted by multiplicity for a polynomial is equal to its degree.</li> <li>Some polynomial equations, including quadratics, may have no real solutions.</li> <li>All complex solutions come in pairs (conjugates) making it necessary that all odd polynomial functions have at least one real solution.</li> </ul> </li> <li>Explain the relationship between the number of real (and complex) solutions and the graph of a polynomial equation.</li> <li>Solve equations and inequalities numerically, graphically, and algebraically, with and without the use of technology, making connections between solution strategies.</li> <li>Solve power equations with integer exponents, <math>ax^n = b</math> algebraically, graphically, and using technology.<br/><i>Power equations can represent area or volume; polynomial equations can represent projectile height, profit, or revenue.</i></li> <li>Solve radical equations numerically, algebraically, and graphically, with and without the use of technology.</li> <li>Use the factored form of a polynomial to determine its real roots.</li> <li>Consider domain restrictions (asymptotes or undefined values) when finding solutions of rational and radical equations.</li> <li>Know which operations on an equation may produce an equation with the same solutions, and which produce an equation with fewer or more solutions (or extraneous solutions).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|  |   |
|--|---|
| <p>b. Solve single variable quadratic equations and inequalities over the complex numbers; graph real solution sets on a number line.</p>            | <ul style="list-style-type: none"> <li>• Solve quadratic equations and inequalities using factoring, completing the square, and the quadratic formula.</li> <li>• Use a calculator to approximate the solutions of a quadratic equation related to an inequality and as an aid in graphing.</li> <li>• Select and explain a method of solution (e.g., exact vs. approximate) that is effective and appropriate for a given problem.</li> <li>• Determine a single variable quadratic equation given its solutions.</li> <li>• Recognize that complex solutions come in conjugate pairs of the form <math>a + bi</math> and <math>a - bi</math> when the coefficients of a quadratic equation are real numbers.</li> </ul> |
| <p>c. Use the discriminant, <math>D = b^2 - 4ac</math>, to determine the nature of the solutions of the equation <math>ax^2 + bx + c = 0</math>.</p> | <ul style="list-style-type: none"> <li>• Describe how the discriminant, <math>D = b^2 - 4ac</math>, indicates the nature of the solutions of the equation <math>ax^2 + bx + c = 0</math>.<br/><i>The solutions are real and distinct if <math>D &gt; 0</math>; real and equal if <math>D = 0</math>; and complex conjugates if <math>D &lt; 0</math>.</i></li> </ul>  |
| <p>d. Graph the solution set of a two-variable quadratic inequality in the coordinate plane.</p>   | <ul style="list-style-type: none"> <li>• Students might also be asked to write or identify the quadratic inequality from the graph.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|   |   |
|---|---|
| <p>e. Rewrite nonlinear equations and inequalities to express them in multiple forms in order to facilitate finding a solution set or to extract information about the relationships or graphs indicated.</p> | <ul style="list-style-type: none"><li>• Inequalities and equations that do not represent functions may be included.<br/>(e.g. horizontal parabolas)<br/><i>Example:</i> Explain how to find the x-intercept for the equation <math>x = y^2 - 6y + 8</math>.<br/><br/><i>Example:</i> For <math>y + 1 \geq (x - 3)^2</math> find the vertex and sketch a graph.</li><li>• Inequalities and equations in one variable will be included.<br/><i>Example:</i> <math>x^2 + 8x + 7 = 0 \Leftrightarrow (x + 7)(x + 1) = 0 \Leftrightarrow x = -7</math> or <math>x = -1</math></li><li>• Rewriting or solving a formula in several variables for one variable in terms of the others will be included.<br/><i>Example:</i> Solve <math>V = \frac{1}{3}\pi r^2 h</math> for r or h.<br/><i>Example:</i> Solve <math>A = \pi r^2 + 2\pi rh</math> for r or h.</li></ul> |
|---|---|

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Core: Polynomial and Rational Functions  
Priority: 30%**

Successful students will be able to use tables, graphs, verbal statements and symbols to represent and analyze quadratic, rational, and higher-order polynomial functions. They will be able to recognize and solve problems that can be modeled using these functions. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples  |
|--|--|
| <b>P1. Quadratic functions</b>   |  |
| <p>a. Determine key characteristics of quadratic functions and their graphs.</p> | <ul style="list-style-type: none"> <li>• Key characteristics include domain and range, vertex, minimum/maximum, intercepts, axis of symmetry, and end behavior.</li> <li>• Recognize that except when <math>a = 0</math>, the graph of <math>f(x) = ax^2 + bx + c</math> is a parabolic curve that always crosses the y-axis but may or may not cross the x-axis, and which opens up when <math>a &gt; 0</math> and down when <math>a &lt; 0</math>.</li> <li>• Recognize the relationship between the intercepts and the factors of a quadratic function.</li> <li>• Recognize <math>x = \frac{-b}{2a}</math> as a way of determining the x-coordinate of the vertex for a parabola in the form <math>y = ax^2 + bx + c</math>.</li> <li>• Recognize the relationship between complex solutions of a quadratic equation and the characteristics and position of its graph. (i.e. If the graph of a function has no x-intercepts, then the function has no real roots.)</li> </ul> <p><i>Example:</i> Use different forms of the function to extract information:</p> <p style="margin-left: 40px;"><math>y = x^2 - 6x + 8</math> makes the y-intercept obvious,</p> <p style="margin-left: 40px;"><math>y = (x - 2)(x - 4)</math> provides access to the zeros, and</p> <p style="margin-left: 40px;"><math>y = (x - 3)^2 - 1</math> makes it easy to find the vertex and sketch the graph.</p> |

## Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|  |   |
|--|---|
| <p>b. Represent quadratic functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>             | <ul style="list-style-type: none"> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |
| <p>c. Describe and represent the effect that changes in the parameters of a quadratic function have on the shape and position of its graph.</p>    | <ul style="list-style-type: none"> <li>Transform the parent quadratic function, <math>y = x^2</math>, to translate, reflect, rotate, stretch or compress the graph.</li> <li>Investigate the changes that occur when the coefficients, including the constant are changed in a quadratic function in the form,<br/><math>f(x) = ax^2 + bx + c</math>.</li> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |
| <p>d. Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.</p> | <ul style="list-style-type: none"> <li>Recognize situations in which quadratic models are appropriate; create and interpret quadratic models to answer questions about those situations.</li> <li>This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul> <p><i>Example:</i> Determine the height of an object above the ground <math>t</math> seconds after it has been thrown upward at an initial velocity of <math>v_0</math> feet per second from a platform <math>d</math> feet above the ground.</p> <p><i>Example:</i> Determine the relationship between the length of a side of a cube and its surface area.</p> <ul style="list-style-type: none"> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

| P2. Higher-order polynomial and rational functions   |   |
|--|---|
| <p>a. Determine key characteristics of power functions in the form <math>f(x) = ax^n</math>, <math>a \neq 0</math>, for positive integral values of <math>n</math> and their graphs.</p> | <ul style="list-style-type: none"> <li>• Recognize the basic power functions as the parent functions for many types of polynomials. For a polynomial function that results from transformations on a power function, that particular power function would be considered the parent function of that polynomial, such as <math>y = x^3</math> is the parent function of <math>y = 3(x - 2)^3 - 1</math>.</li> </ul> <p><i>Example:</i> Determine the parent function from a graph of a polynomial that is the transformation of a power function (translated, stretched, compressed, reflected).</p> <ul style="list-style-type: none"> <li>• Identify the type of symmetry and relate to odd/even exponents.</li> </ul>   |
| <p>b. Determine key characteristics of polynomial functions and their graphs.</p>  | <ul style="list-style-type: none"> <li>• Key characteristics include domain and range, intercepts, end behavior, and degree.<br/><i>Know that polynomial functions of degree <math>n</math> have the general form <math>f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0</math> where <math>n</math> is a positive integer. The degree of the polynomial function is the largest power of its terms for which the coefficient is non-zero. The leading coefficient is the coefficient of the term of highest degree.</i> <ul style="list-style-type: none"> <li>○ Identify and use the degree of a polynomial or its factors to interpret characteristics of the function or its graph.</li> <li>○ Note that every polynomial function of odd degree has at least one real zero.</li> <li>○ Understand the concept of the <i>multiplicity</i> of a root of a polynomial equation and its relationship to the graph of the related polynomial function.</li> </ul> </li> </ul> |
| <p>c. Represent polynomial functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>  | <ul style="list-style-type: none"> <li>• Identify or write a polynomial function of a given degree.</li> <li>• Decide if a given graph or table of values suggests a higher-order polynomial function.</li> </ul>   |
| <p>d. Determine key characteristics of simple rational functions and their graphs.</p>   | <ul style="list-style-type: none"> <li>• Key characteristics include domain and range, intercepts, types of symmetry, horizontal and vertical asymptotes, and end behavior.</li> <li>• Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form <math>f(x) = ax^n</math> (<math>a \neq 0</math>) for negative integral values of <math>n</math>.</li> </ul>  |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|   |   |
|---|---|
| <p>e. Represent simple rational functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>                            | <ul style="list-style-type: none"> <li>Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form <math>f(x) = ax^n</math> (<math>a \neq 0</math>) for negative integral values of <math>n</math>.</li> </ul> <p><i>Example:</i> Graph <math>f(x) = \frac{x}{x^2 - x} + 5</math>.</p> <ul style="list-style-type: none"> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>   |
| <p>f. Recognize, express, and solve problems that can be modeled using polynomial and simple rational functions. Interpret their solutions in terms of the context.</p> | <ul style="list-style-type: none"> <li>This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul> <p><i>Example:</i> The volume of a cylinder whose radius is 5 more than twice the radius, <math>r</math>, of a given sphere and whose height is twice its own radius is <math>6,750\pi</math>. Determine the radius of the given sphere.</p> <ul style="list-style-type: none"> <li>Solve problems involving direct, inverse, and joint variations.</li> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Core: Exponential Functions  
Priority: 20%**

Successful students will be able to use tables, graphs, verbal statements and symbols to represent, analyze, model, and interpret graphs of exponential functions. While some facility with the properties of logarithms may be helpful, it is not required on the core exam. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples   |
|--|---|
| <b>X1. Exponential functions</b>   |   |
| a. Determine key characteristics of exponential functions and their graphs.  | <ul style="list-style-type: none"> <li>Key characteristics include domain and range, intercepts, and end behavior.</li> </ul>   |
| b. Represent exponential functions using tables, graphs, verbal statements, and equations. Represent exponential equations in multiple forms. Translate among these representations. | <ul style="list-style-type: none"> <li>Know that exponential functions have the general form <math>f(x) = ab^x + c</math>, <math>b &gt; 0</math>, and <math>b \neq 1</math>.</li> <li>Distinguish between and graph exponential functions that are growth functions, such as <math>f(x) = 3 \cdot 2^x</math> where <math>x &gt; 0</math>, or decay functions, such as <math>f(x) = 3 \cdot 2^{-x}</math> where <math>x &gt; 0</math>.</li> <li>Decide if a given graph or table of values represents an exponential function.<br/><i>Be aware that it can be very difficult to distinguish exponential graphs from graphs of other functions, particularly polynomial functions, over small regions or particular subsets of their domains. Sometimes the context of an underlying situation can suggest a likely type of function model.</i></li> <li>Translate from exponential to logarithmic form and vice versa (e.g. <math>a^b = c \rightarrow \log_a c = b</math> )</li> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

## Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|  |   |
|--|---|
| <p>c. Describe and represent the effect that changes in the parameters of an exponential function have on the shape and position of its graph.</p>   | <ul style="list-style-type: none"> <li>• Explain or illustrate the effect that changes in a parameter (<math>a</math> or <math>c</math>) or the base (<math>b</math>) have on the graph of the exponential function <math>f(x) = ab^x + c</math>.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |
| <p>d. Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.</p> | <ul style="list-style-type: none"> <li>• A logarithm is an exponent that depends on the base used. Logarithms provide an efficient method for solving problems with variable exponents. Limitation: Logarithms may be used to solve problems but will not be used in the text of an item.<br/><i>If <math>x^y = z</math>, <math>x &gt; 0</math>, <math>x \neq 1</math>, and <math>z &gt; 0</math>, then <math>y</math> is the logarithm to the base <math>x</math> of <math>z</math>. The equation <math>y = \log_x z</math> is one of three equivalent forms of expressing the relation <math>x^y = z</math> (the other being <math>x = \sqrt[y]{z}</math>).</i></li> <li>• Exponential problems in which the student must determine the exponent may also be solved using graphing techniques.</li> <li>• Exponential growth functions using base <math>e</math> may be included (e.g. <math>A = Pe^{rt}</math>, or <math>y = Ce^{kt}</math>). Formulas will be provided in these cases.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.<br/><i>Example:</i> If a culture had 500 cells at noon and 600 cells at 1:00 PM, what is the approximate doubling time of the cell population? Approximately how many cells will there be at 4:00 PM?<br/><i>Example:</i> If radioactive iodine 123 has a half-life of 8 days, what percentage of an original dose will remain in a patient's body twelve hours after a medical test has been performed in which iodine 123 was administered?</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Core: Function Operations and Inverses  
Priority: 15%**

Successful students will be able to perform function operations of addition, subtraction, multiplication, division, and composition and to combine several functions defined over restricted domains to form a piecewise-defined function. They will be able to determine, graph and analyze the inverse of a function and use composition to determine whether two functions are inverses. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections.

| Content Benchmarks  | Explanatory Comments and Examples   |
|---|---|
| <b>F1. Function operations</b>  |   |
| <p>a. Combine functions by addition, subtraction, multiplication, and division.</p>                       | <p><i>Example:</i> If <math>f(x) = 3x^4 - 5x^3 + 3</math>, and <math>g(x) = x^4 - 3x^3 + 2x^2 + 5</math>, then <math>f(x) - g(x) =</math></p> $\begin{aligned} & (3x^4 - 5x^3 + 3) - (x^4 - 3x^3 + 2x^2 + 5) \\ &= (3x^4 - x^4) + (-5x^3 - (-3x^3)) - 2x^2 + (3 - 5) \\ &= 2x^4 - 2x^3 - 2x^2 - 2 \end{aligned}$ <p><i>Example:</i> If <math>h(x) = x^4 + x^3 + 2x^2 + 6x + 4</math> and <math>j(x) = x + 1</math>, then</p> $h(x) \div j(x) = x^3 + 2x + 4, x \neq -1.$ <p><i>Sample Solution:</i> Using synthetic division</p> $\begin{array}{r rrrrr} -1 & 1 & 1 & 2 & 6 & 4 \\ & & -1 & 0 & -2 & -4 \\ \hline & 1 & 0 & 2 & 4 & 0 \end{array}$ <p><i>Example:</i> If <math>r(x) = x^2 + 3x - 1</math> and <math>s(x) = x + 3</math>, then</p> $r(x) \cdot s(x) = (x^2 + 3x - 1)(x + 3) = x^3 + 6x^2 + 8x - 3$ |
| <p>b. Determine the composition of two functions, including any necessary restrictions on the domain.</p> | <p><i>Example:</i> If <math>f(x) = 3x - 2</math>, and <math>g(x) = \sqrt{x + 1}</math> find <math>f(g(x))</math> and <math>g(f(x))</math>. Include domain restrictions on each.</p> <p><i>Example:</i> If <math>f(x) = \frac{3}{5} + \frac{2}{5x}</math>, and <math>g(x) = \frac{2}{5x - 3}</math>, determine whether or not <math>f</math> and <math>g</math> are inverses and explain how you know.</p>   |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

| <b>F2. Inverse functions</b>   |  |
|--|--|
| <p>a. Describe the conditions under which an inverse relation is a function.</p> | <ul style="list-style-type: none"> <li>• Consider graphic conditions for an inverse relation to be a function.</li> <li>• Use the horizontal line test to determine whether the inverse of a function is also a function. Consider domain restrictions for existence of an inverse function.</li> <li>• Recognize that the inverse of a quadratic function is a function only when its domain is restricted.</li> </ul>  |
| <p>b. Determine and graph the inverse relation of a function.</p>                | <ul style="list-style-type: none"> <li>• Include inverses which may not be functions.</li> </ul> <p><i>Example:</i> The inverse relation of <math>y = x^2</math> is <math>y = \pm\sqrt{x}</math>. Explain why an inverse function would be only either the positive or the negative part of the graph.</p> <ul style="list-style-type: none"> <li>• Explain why the graphs of a function and its inverse are reflections of each other over the line <math>y = x</math>.</li> <li>• Show that when the inverse of a function is a function <math>f^{-1}(f(x)) = x</math> and <math>f(f^{-1}(x)) = x</math></li> </ul> <p><i>Example:</i> Determine the inverse relation for <math>f(x) = 3x^2 + 5</math>.</p> <p><i>Example:</i> Determine <math>g(x)</math> when <math>g^{-1}(x) = 9x - 81</math>.</p> <ul style="list-style-type: none"> <li>• Represent the inverse of an exponential function graphically but not algebraically.</li> </ul> <p><i>Example:</i> Graphically represent <math>f^{-1}(x)</math> when <math>f(x) = 2^x - 7</math>.</p> <ul style="list-style-type: none"> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

| <b>F3. Piecewise-defined functions</b>   |  |
|--|--|
| <p>a. Determine key characteristics of absolute value, step, and other piecewise-defined functions.</p>  | <ul style="list-style-type: none"> <li>• Key characteristics include vertices, intercepts, end behavior, slope of linear sections, and discontinuities.</li> <li>• Determine the vertex, slope of each branch, intercepts, and end behavior of an absolute value graph.</li> </ul>   |
| <p>b. Represent piecewise-defined functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>   | <ul style="list-style-type: none"> <li>• Interpret the algebraic representation of a piecewise defined function; graph it over the appropriate domain.</li> <li>• Write an algebraic representation for a given piecewise defined function.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>                 |
| <p>c. Recognize, express, and solve problems that can be modeled using absolute value, step, and other piecewise-defined functions. Interpret their solutions in terms of the context.</p> | <ul style="list-style-type: none"> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> <li>• Applications may include postage rates, salary increases, or cellular telephone charges.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Data and Statistics**

Successful students will be able to analyze, interpret, compare, and compute with summary statistics for sets of data. Analysis of bivariate data includes the determination and interpretation of regression lines and correlation coefficients. While some important components in the study of data and statistics, such as sampling techniques, question formulation, and experiment design are addressed when possible on this module of the Algebra II End-of-Course Exam, those topics will be expected to be assessed in more depth in the classroom. This module includes a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks  | Explanatory Comments and Examples   |
|---|---|
| <b>S1. Summarization and comparison of data sets</b>                    |   |
| a. Summarize and compare data sets using statistical methods.           | <ul style="list-style-type: none"> <li>• For univariate categorical data, use percentages and proportions (relative frequencies).</li> <li>• For bivariate categorical data, use conditional (row or column) percentages or proportions.</li> <li>• For univariate quantitative data, use measures of center (mean and median) and measures of spread (percentiles, quartiles, interquartile range, and standard deviation).</li> <li>• Graphically represent measures of center and spread (variability) for quantitative data.</li> <li>• Use box plots to compare key features of data distributions.</li> </ul>   |
| b. Determine, use, and identify potential misuses of weighted averages. | <ul style="list-style-type: none"> <li>• Identify and interpret common instances of weighted averages.</li> <li>• Analyze variation in weighted averages and distinguish change due to weighting from changes in the quantities measured.</li> <li>• Contexts may include composite grades, stock market indexes, consumer price index, or unemployment rate.</li> </ul> <p><i>Example:</i> Suppose a company employed 100 women with average annual salaries of \$20,000 and 500 men with average salaries of \$40,000. After a change in management, the company employed 200 women and 400 men. To correct past inequities, the new management increased women's salaries by 25% and men's salaries by 5%. Explain why, despite these increases, the company's average salary declined by almost 1%.</p> |

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|  |   |
|--|---|
| <p>c. Use a computer or calculator to find a linear regression equation (least squares line) as a model for data that suggest a linear trend, and determine the correlation coefficient.</p> | <ul style="list-style-type: none"> <li>• Regression can be used to summarize bivariate quantitative data.</li> <li>• Know that the least squares line passes through the point <math>(\bar{x}, \bar{y})</math>, where the mean of the x-coordinates of the data points is <math>\bar{x}</math> and the mean of the y-coordinates of the data points is <math>\bar{y}</math>.</li> <li>• Recognize correlation as a number between <math>-1</math> and <math>+1</math> that measures the direction and strength of linear association between two variables.</li> </ul>  |
| <p><b>S2. Interpretation and communication through data</b></p>  |   |
| <p>a. Analyze the strength of the linear relationship indicated by the regression line.</p>  | <ul style="list-style-type: none"> <li>• Common terms such as least squares line, regression line, and line of best or good fit will be used in the problems.</li> <li>• Use the relationship among the standard deviation, correlation coefficient, and slope of the regression line to assess the strength of linear association suggested by an underlying scatter plot.</li> <li>• Interpret the slope of a linear trend line in terms of the data being studied.</li> <li>• Identify the effect of outliers on the position and slope of the regression line.</li> <li>• Analyze a residual plot to informally assess whether a line provides a good model for a set of data.</li> </ul>   |
| <p>b. Interpret data and communicate conclusions effectively.</p>  | <ul style="list-style-type: none"> <li>• Interpret, interpolate, and judiciously extrapolate from graphs and tables.</li> <li>• Use the basic properties of confidence intervals to make simple predictions and answer questions about statistical data.</li> <li>• State conclusions in terms of the question(s) being investigated.</li> <li>• Use appropriate statistical language when reporting on plausible answers that go beyond the data actually observed.</li> <li>• Interpret oral, written, graphic, pictorial, or multi-media reports on data.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|   |   |
|---|---|
| <p>c. Make judgments regarding accuracy, reasonableness, and bias in the use of data.</p>                   | <ul style="list-style-type: none"><li>• Identify and explain misleading uses of data by considering the completeness and source of the data, the design of the study, and the way the data are analyzed and displayed.</li><li>• Recognize the difference between correlation or association and causation.</li></ul> <p><i>Example:</i> Determine whether the height or area of a bar graph is being used to represent the data; evaluate whether the scales of a graph are consistent and appropriate or whether they are being adjusted to alter the perception conveyed by the information.</p>   |
| <p>d. Critique and justify various methods of sampling and data collection used in real world problems.</p> | <ul style="list-style-type: none"><li>• Some possible real world applications may be clinical trials in medicine, an opinion poll, or a report on the effect of smoking on health.</li><li>• Distinguish between random sampling from a population in sample surveys and random assignment of treatments to experimental units in an experiment:<br/><i>Random sampling is how items are selected from a population so that the sample data can be used to estimate characteristics of the population; random assignment is how treatments are assigned to experimental units so that comparisons among the treatment groups can allow cause and effect conclusions to be made.</i></li></ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Probability**

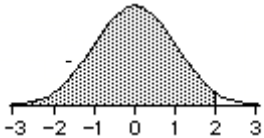
Successful students will be able to quantify the likelihood that an event will occur through combinatorics and other counting principles, relative frequency, distributions, and the comparison of theoretical probability to simulations. Also included are binomial expansion and the relationship to Pascal's triangle and binomial distributions. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples   |
|--|---|
| <b>R1. Permutations, combinations, and probability</b>   |   |
| <p>a. Determine the number of ways events can occur using permutations, combinations, and other systematic counting methods.</p> | <ul style="list-style-type: none"> <li>• Know and apply organized counting techniques such as the Fundamental Counting Principle.<br/><i>The Fundamental Counting Principle is a way of determining the number of ways a sequence of events can take place. If there are <math>n</math> ways of choosing one thing and <math>m</math> ways of choosing a second after the first has been chosen, then the Fundamental Counting Principle says that the total number of choice patterns is <math>n \times m</math>.</i></li> <li>• Simplify the factorial form of the permutation and combination formula to aid computation.</li> <li>• Distinguish between situations where replacement is not permitted and situations that permit replacement, including those involving circular arrangements.</li> <li>• Distinguish between situations where order matters and situations where it does not; select and apply appropriate means of computing the number of possible arrangements of the items in each case.</li> <li>• Use permutations and combinations to determine the number of possible outcomes of an event.</li> <li>• Use factorials to define combinations and permutations and to determine the number of ways an event can occur.</li> </ul> <p><i>Example:</i> Interpret <math>6!</math> as the product <math>6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1</math>; Show that</p> $\frac{15!}{12!} = 15 \cdot 14 \cdot 13 = 2730 .$ |

## Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|  |  |
|--|--|
|  | <p><i>Example:</i> Determine the number of different license plates that can be formed with two letters and three numerals; determine how many are possible if all the letters have to come first; determine how many different subsets are possible for a set of six elements?</p> <p><i>Example:</i> Determine the number of different four-digit numbers that can be formed if the first digit must be non-zero and each digit may be used only once; determine how many are possible if the first digit must be non-zero but digits can be used any number of times.</p> <p><i>Example:</i> Determine the number of ways that first, second, and third place could be awarded in a race having 9 contestants if there are no ties <math>{}_9P_3</math>; determine the number of distinct committees of five people that can be formed from a class of 26 students <math>{}_{26}C_5</math>.</p> |
| <p>b. Relate the expansion of <math>(x + y)^n</math> (i.e., the binomial theorem) with the possible outcomes of a binomial experiment and/or the nth row of Pascal's triangle.</p> | <p><i>Example:</i> A basketball player has an average success rate of 60% on free throws. What is the probability that she will make 10 out of 12 free throws in the next game?</p>  |
| <p>c. Apply probability concepts to calculate the probability of events and to make informed decisions in practical situations.</p>  | <ul style="list-style-type: none"> <li>• Construct and use decision trees.<br/><i>A tree is a connected graph containing no closed loops (cycles).<br/>Note: Tree diagrams can also be used to analyze games such as tic-tac-toe or Nim or to simply organize outcomes.</i></li> <li>• Use the concepts of conditional probability to calculate simple probabilities.</li> <li>• Represent and analyze probabilities of independent events (e.g., repeated tossing of a coin, or throwing dice).</li> </ul>  |
| <p>d. Analyze and interpret actual data to estimate probabilities and predict outcomes, including those involving relative frequency.</p>  | <ul style="list-style-type: none"> <li>• Recognize and use relative frequency as an estimate for probability.</li> <li>• If an action is repeated <math>n</math> times and a certain event occurs <math>b</math> times, the ratio <math>\frac{b}{n}</math> is called the relative frequency of the event occurring.</li> </ul> <p><i>Example:</i> In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks?</p>   |

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|   |  |
|---|--|
| <p>e. Compare theoretical probabilities with the results of simple experiments (e.g., tossing number cubes, flipping coins, spinning spinners).</p> | <ul style="list-style-type: none"> <li>Explain how the Law of Large Numbers defines the relationship between experimental and theoretical probabilities.<br/><i>The law of large numbers indicates that if an event of probability <math>p</math> is observed repeatedly during independent repetitions, the ratio of the observed frequency of that event to the total number of repetitions approaches <math>p</math> as the number of repetitions becomes arbitrarily large.</i></li> <li>Use simulations to estimate probabilities.</li> <li>Use theoretical probability to determine the most likely result if an experiment is repeated a large number of times.</li> </ul>  |
| <p>f. Compute and graph cumulative frequencies.</p>   | <ul style="list-style-type: none"> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>   |
| <p><b>R2. Probability distributions</b></p>   |  |
| <p>a. Identify and distinguish between discrete and continuous probability distributions.</p>   | <p><i>Example:</i> Compare and contrast the binomial distribution (discrete) and the normal distribution (continuous).</p>   |
| <p>b. Identify the principal characteristics of the normal distribution and use them to estimate probabilities.</p>                                 | <ul style="list-style-type: none"> <li>Understand common examples that fit the normal distribution (e.g., height, weight) and examples that do not (salaries, housing prices, size of cities) and explain distinguishing characteristics of each.<br/><i>The normal (or Gaussian) distribution is actually a family of mathematical distributions that are symmetric in shape with scores more concentrated in the middle than in the tails. They are sometimes described as bell shaped. Normal distributions may have differing centers (means) and scale (standard deviation). The standard normal distribution is the normal distribution with a mean of zero and a standard deviation of one. In this distribution, approximately 68% of the data lie within one standard deviation of the mean, 95% within two, etc.</i></li> </ul> <div style="text-align: right;">  </div> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|   |   |
|---|---|
| <p>c. Identify and describe the key characteristics of and create frequency distributions of both discrete and continuous data.</p> | <ul style="list-style-type: none"><li>• Describe key characteristics (e.g., shape, symmetry/skewness, typical value, and/or spread) of a frequency distribution.<br/><i>A frequency distribution shows the number of observations falling into each of several ranges of values; if the percentage of observations is shown, the distribution is called a relative frequency distribution. Both frequency and relative frequency distributions are portrayed through tables, histograms, or broken-line graphs.</i></li><li>• Use a probability distribution to assess the likelihood of the occurrence of an event.</li><li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li></ul> |
|---|---|

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Logarithmic Functions**

Successful students will be able to define, represent, and model using logarithmic functions. Recognition of the inverse relationship between logarithmic and exponential functions is essential to this concept. They will apply the laws of logarithms, solve logarithmic equations, and use logarithms to solve exponential equations. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks  | Explanatory Comments and Examples  |
|---|--|
| <b>L1. Logarithmic expressions and equations</b>  |  |
| a. Apply the properties of logarithms and use them to manipulate logarithmic expressions.   | <ul style="list-style-type: none"> <li>• Represent logarithmic expressions in exponential form and exponential expressions in logarithmic form.</li> <li>• Understand that a logarithm is an exponent that depends on the base used.</li> <li>• The properties of logarithms include those related to powers, products, quotients, and changing the base.</li> </ul> |
| b. Solve logarithmic equations, paying attention to the possibility of extraneous roots.  | <p><i>Example:</i> If <math>\log(x - 3) + \log(x + 2) = \log(14)</math>, then</p> $\log(x^2 - x - 6) = \log(14)$ $x^2 - x - 6 = 14$ $x^2 - x - 20 = 0$ $x = 5 \text{ or } -4$ <p>However, all solutions must be greater than 3 because of domain restrictions on the original logarithmic functions, and the only solution is <math>x = 5</math>.</p>                |
| <b>L2. Logarithmic functions</b>  |  |
| a. Determine key characteristics of logarithmic functions.  | <ul style="list-style-type: none"> <li>• Key characteristics include domain, range, asymptotes, and end behavior.</li> </ul>   |
| b. Represent logarithmic functions using tables, graphs, verbal statements, and equations. Translate among these representations. | <ul style="list-style-type: none"> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>                     |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|   |   |
|---|---|
| <p>c. Describe the effect that changes in the parameters of a logarithmic function have on the shape and position of its graph.</p>                                 | <ul style="list-style-type: none"><li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li></ul>  |
| <p>d. Recognize, express, and solve problems that can be modeled using logarithmic functions. Interpret their solutions in terms of the context of the problem.</p> | <ul style="list-style-type: none"><li>• Use the properties of logarithms to solve problems involving logarithmic and exponential functions, including problems of growth and decay, chemical pH, and sound or earthquake intensity.</li><li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li><li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li></ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Trigonometric Functions**

Successful students will be able to recognize and model periodic phenomena using trigonometric functions. They will understand the relationship among the unit circle, the geometric definitions of sine and cosine, and the degree and radian measures of angles and will apply this understanding to graph trigonometric functions, determine key characteristics of the functions and their graphs, and describe the effect of transformations on both the symbolic and graphical representations of the functions. There are a variety of types of assessment items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples   |
|--|---|
| <b>T1. Trigonometric Functions</b>   |   |
| a. Recognize periodic phenomena and determine key characteristics of such phenomena.   | <ul style="list-style-type: none"> <li>Key characteristics include the period between repetitions, the frequency of the repetitions, and the range of values.</li> </ul> <p>Periodic phenomena include sound waves, length of daylight, and situations involving circular motion.</p>   |
| b. Use the relationship of the sine and cosine functions to a central angle of the unit circle to determine the exact trigonometric ratio of angles on the unit circle. ( $0^\circ$ to $360^\circ$ , 0 to $2\pi$ ) | <ul style="list-style-type: none"> <li>Interpret the sine, cosine, and tangent functions corresponding to a central angle of the unit circle in terms of horizontal and vertical sides of right triangles based on that central angle.</li> </ul>   |
| c. Explain and use both degree and radian measure for angles.  | <ul style="list-style-type: none"> <li>Convert between the degree and radian measure of an angle.</li> </ul>  |
| d. Represent trigonometric functions using tables, graphs, verbal statements, and equations. Translate among these representations.  | <ul style="list-style-type: none"> <li>Use the unit circle to extend the domain of the sine and cosine functions to the set of real numbers.</li> <li>For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |
| e. Determine key characteristics of trigonometric functions and their graphs.  | <ul style="list-style-type: none"> <li>Key characteristics include zeros, amplitude, period, phase shift, vertical shift, and asymptotes.</li> </ul>  |

## Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples January 2010

|  |   |
|--|---|
| <p>f. Describe the effect that changes in the parameters of an equation of a trigonometric function in the form, <math>f(x) = A \sin B(x - C) + D</math> (or the similar cosine function) have on the shape and position of its graph.</p> | <ul style="list-style-type: none"> <li>• Identify changes in the period, amplitude, vertical shift, and phase shift for the function.</li> <li>• Identify the zeros of a function that has a vertical shift of 0.</li> <li>• Identify and describe the transformations (vertical shifts, phase shifts, stretches and compressions) that occur when parameters are changed in trigonometric functions.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>  |
| <p>g. Recognize, express, and solve problems that can be modeled using trigonometric or other periodic functions.</p>  | <p><i>Example:</i> Explain the difference between frequency modulation used in FM radio signals and amplitude modulation used in AM radio signals.</p> <ul style="list-style-type: none"> <li>• Use Pythagorean identities for sine, cosine, and tangent to solve trigonometric equations and problems.</li> <li>• Use inverse trigonometric functions to solve equations and problems.</li> <li>• Solve problems in context that rely on trigonometric relationships.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Matrices**

Successful students will be able to compute with and use matrices to organize information, solve systems of equations, and perform transformations of geometric figures. They will use and interpret matrix notation to represent a vector and perform operations on vectors and matrices. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve contextual problems.

| Content Benchmarks  | Explanatory Comments and Examples  |
|---|--|
| <b>M1. Matrix arithmetic</b>  |  |
| a. Perform addition, subtraction, and scalar multiplication of matrices.  | <ul style="list-style-type: none"> <li>• Recognize that matrix addition is associative and commutative.</li> <li>• Know and fluently use matrix notation for rows, columns, and entries of cells.</li> </ul>   |
| b. Perform matrix multiplication.   | <ul style="list-style-type: none"> <li>• Describe the characteristics of matrices that can be multiplied and those that cannot.</li> <li>• Recognize that matrix multiplication is not commutative and provide examples of square matrices <math>A</math> and <math>B</math> such that <math>AB \neq BA</math>.</li> <li>• Recognize that matrix multiplication is distributive with respect to addition.</li> <li>• Apply the associative property of matrix multiplication.<br/><i>The associative property of matrix multiplication states that if there are three matrices, <math>A</math>, <math>B</math>, and <math>C</math> such that <math>AB</math> and <math>BC</math> are defined, then <math>(AB)C</math> and <math>A(BC)</math> are defined and <math>(AB)C = A(BC)</math>.</i></li> <li>• Know and use the algorithm for matrix multiplication.</li> </ul> |
| <b>M2. Solving systems of equations using matrices</b>  |  |
| a. Find the determinant of a 2x2 or 3x3 matrix.   |  |
| b. Determine the inverse of a 2x2 or 3x3 matrix or indicate that no inverse exists.                                 | <ul style="list-style-type: none"> <li>• Know and use the definition and properties of the identity matrix.</li> <li>• Find the inverse of a 2x2 matrix if the inverse exists.</li> <li>• Use row reduction to find the inverse of a 3x3 matrix when it exists.</li> </ul>   |
| c. Represent 2-variable and 3-variable systems of linear equations using matrices and use them to solve the system. | <ul style="list-style-type: none"> <li>• Translate between equation and matrix notation of a system of linear equations.</li> </ul>  |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|  |  |
|--|--|
|  | <ul style="list-style-type: none"> <li>Use both technology and by-hand methods, including row reduction, substitution, and elimination, to solve systems involving 3 equations.</li> <li>3x3 systems to be solved by hand should have integer or simple fractional solutions and small integral coefficients.</li> </ul>   |
| d. Solve a matrix equation.  | <ul style="list-style-type: none"> <li>Use the inverse of a matrix to solve a matrix equation.</li> </ul>  |
| <b>M3. Matrix transformations</b>  |  |
| a. Use matrix tools to represent and transform geometric objects in the coordinate plane.                                | <ul style="list-style-type: none"> <li>Use 2x2 matrices to represent transformations in the coordinate plane.<br/> <i>Example:</i> <math>\begin{bmatrix} -1 &amp; 0 \\ 0 &amp; -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}</math><br/> represents the rotation of the point A(3, 1) 180° about the origin.</li> <li><i>Example:</i> <math>\begin{bmatrix} -1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}</math><br/> represents the reflection of the point A(3, 1) over the y-axis.</li> </ul> |
| <b>M4. Vectors</b>   |  |
| a. Represent vectors as matrices in two dimensions.  | <ul style="list-style-type: none"> <li>Use either horizontal or vertical representation of vectors, with an understanding of both.</li> <li>Use vectors to represent and transform geometric objects in the coordinate plane.</li> <li>Know the relationship between the ordered pair representation of a vector and its graphical representation.</li> </ul>  |
| b. Add, subtract, and compute the dot product of two-dimensional vectors; multiply a two-dimensional vector by a scalar. | <ul style="list-style-type: none"> <li>Represent addition, subtraction, and scalar multiplication of vectors graphically.</li> <li>Use the dot product of two vectors to determine whether they are normal (perpendicular) to each other.</li> </ul>   |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Conic Sections**

Successful students will be able to represent, analyze, and model using the circle, ellipse, and hyperbola. In addition parabolas with a horizontal axis are included. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

| Content Benchmarks   | Explanatory Comments and Examples  |
|--|--|
| <b>C1. Conic Sections</b>  |  |
| <p>a. Identify a parabola, circle, ellipse, or hyperbola from its equation, description, or key characteristics.</p>   | <ul style="list-style-type: none"> <li>• Key characteristics of the graph include any of the following that apply: vertices, major and minor axes and their lengths, foci, directrix, eccentricity, asymptotes, and center.</li> <li>• Determine which conic section is represented by a given quadratic equation.</li> </ul>  |
| <p>b. Represent conic sections whose axes are parallel to the x- and y-axes using graphs, verbal statements, and equations. Translate among these representations. Represent the equations of conic sections in multiple forms to extract information about the parabola, circle, ellipse, or hyperbola.</p> | <ul style="list-style-type: none"> <li>• Determine the graph of a conic section whose axes are parallel to the x- and y-axes from its equation, an equation from its graph, and either the equation or graph from verbal information regarding its position and/or characteristics (e.g. vertices, foci, radius, or major and minor axis length).</li> <li>• Translate from the general form for conic sections, <math>Ax^2 + By^2 + Cx + Dy + E = 0</math>, to the standard form for graphic analysis.</li> <li>• Communicate understanding of the relationship between the standard algebraic forms and graphical characteristics for parabolas, circles, ellipses, and hyperbolas.</li> <li>• The standard form of an ellipse <math>\frac{(x-h)^2}{16} + \frac{(y-k)^2}{9} = 1</math> identifies the center as <math>(h, k)</math> and major axis as the x-axis; it provides sufficient information to determine the location of the vertices and foci and the eccentricity.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

|  |  |
|--|--|
| <p>c. Describe the effect that changes in the parameters of a particular conic section have on its shape and position.</p>                                   | <ul style="list-style-type: none"> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul> <p><i>Assessment Limitations:</i> Parabolas should be limited to those defined by reference to focus, directrix, and/or eccentricity or to parabolas with horizontal axes, i.e. those that can be represented in the form, <math>x = ay^2 + by + c</math>. Parabolas in the form <math>y = ax^2 + bx + c</math> should be assessed in the core test in Polynomial and Rational Functions.</p> |
| <p>d. Recognize, express, and solve problems that can be modeled using conic sections. Interpret their solutions in terms of the context of the problem.</p> | <ul style="list-style-type: none"> <li>• Some real world applications using conic sections may include satellite dish design, planet or satellite orbits, whispering galleries, tracking systems, parabolic mirrors, telescopes, or long range navigational systems.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> <li>• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).</li> </ul>                              |

**Achieve ADP Algebra II End-of-Course Exam Content Standards  
with Comments & Examples  
January 2010**

**Module: Sequences and Series**

This module addresses the patterns in arithmetic and geometric sequences and series. Students are expected to apply the formulas for finding the  $n$ th term of a sequence or series, the  $n$ th partial sum of finite series, and the infinite sum of a geometric series when it exists. General iterative relationships and recursive models are applied to patterns and problems. There will be a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems, where appropriate.

| Content Benchmarks   | Explanatory Comments and Examples  |
|--|--|
| <b>I 1. Arithmetic and geometric sequences and series</b>  |  |
| a. Represent the general term of an arithmetic or geometric sequence and use it to generate the sequence or determine the value of any particular term.  | <ul style="list-style-type: none"> <li>• Use and interpret sigma notation, <math>\Sigma</math>, to indicate summation.</li> <li>• Recognize that the sequence defined by:<br/>First term = 5; Each term after the first is 6 more than the preceding term as the sequence whose first seven terms are 5, 11, 17, 23, 29, 35, and 41.</li> </ul>  |
| b. Represent partial sums of an arithmetic or geometric sequence and determine the value of a particular partial sum or sum of a finite sequence.  | <ul style="list-style-type: none"> <li>• Use the formulas for sums of finite arithmetic and geometric series to find partial sums or the sum of <math>n</math> terms.</li> </ul>   |
| c. Recognize when an infinite geometric sum can be determined and determine the sum when possible.   | <ul style="list-style-type: none"> <li>• Determine that the sum exists when the common ratio, <math>r</math>, is between 1 and -1. (<math> r  &lt; 1</math>)</li> <li>• To find the infinite sum, when it exists, use the formula<br/><math display="block">S_{\infty} = \frac{a}{1-r}</math>, where <math>a</math> represents the first the first term of the geometric series and <math>r</math> is the common ratio.</li> </ul> |
| d. Convert the recursive model for linear growth ( $a_1 = a, a_{n+1} = a_n + d$ , where $a$ is the first term and $d$ is the constant difference) to a closed linear form ( $a_n = a + (n-1)d$ ).  |  |
| e. Convert the recursive model of geometric growth ( $p_1 = a, p_{n+1} = rp_n$ where $a$ is the first term and $r$ is the constant growth rate) to a closed exponential form ( $p_n = ar^{n-1}$ ). |  |

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples

**January 2010**

|   |   |
|---|---|
| <p>f. Recognize, express, and solve problems that can be modeled using a finite geometric series. Interpret their solutions in terms of the context of the problem.</p> | <ul style="list-style-type: none"> <li>• Possible applications are home mortgage problems and other compound interest problems.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul>  |
| <b>12. Other types of iteration and recursion</b>   |   |
| <p>a. Use recursion to generate and describe, analyze, and interpret patterned relationships other than arithmetic or geometric sequences.</p>                          | <ul style="list-style-type: none"> <li>• Analyze and explain the iterative steps in standard algorithms for arithmetic</li> <li>• Use recursion to generate and describe patterned relationships.</li> <li>• Analyze sequences produced by recursive calculations using spreadsheets.</li> </ul> <p><i>Example:</i> The result of repeatedly squaring a number between -1 and 1 appears to approach zero; the result of repeatedly squaring a number less than -1 or greater than 1 appears to continue to increase; determine empirically how many steps are needed to produce 4-digit accuracy in square roots by iterating the operations divide and average.</p> <ul style="list-style-type: none"> <li>• Describe the factorial function or the Fibonacci sequence recursively.</li> </ul> |
| <p>b. Use iterative methods to solve problems.</p>  | <ul style="list-style-type: none"> <li>• Display the effect of iteration using a spreadsheet.</li> </ul> <p><i>Example:</i> Use a spreadsheet to display and compare the effect of compound interest on loans or investments at different rates of interest; identify the diminishing effect of increasing the number of times per year that interest is compounded.</p>  |



## ADP Algebra I and Algebra II End-of-Course Exam Notation Information

The information below is meant to inform teachers, schools, districts, and states about the notation **that students will see on the ADP Algebra I and Algebra II End-of-Course Exams**. It is not meant to exclude acceptable notation from being used in the classroom, but to let teachers know what students should expect to see on the exam. We expect students to be exposed to and use multiple forms of correct notation in their classrooms. In addition, students may answer items using any acceptable form of notation on the exam.

---

### Notation on Both Algebra I and Algebra II End-of-Course Exams

Absolute value functions:  $f(x) = -3|x + 2| + 1$

Set notation:  $\{-1, 0, 4\}$  for solution sets

Negative fractions and rational expressions:  $-\frac{2}{3}$   $-\frac{x+1}{x}$

Monomials involving roots and exponents:  $x^2 y \sqrt[4]{z^3}$   
( $x$  squared multiplied by  $y$  multiplied by the fourth root of  $z$  cubed)

---

### Notation on Algebra II End-of-Course Exams – Core

Piecewise functions:  $f(x) = \begin{cases} x - 2, & x \leq 0 \\ x + 2, & x > 0 \end{cases}$

Greatest Integer Function:  $f(x) = [x]$

Exponential functions of base  $e$ :  $f(x) = 2e^{-0.024x}$

Domain restrictions: When a student is asked only to simplify a rational expression, all expressions are assumed to be defined. If a student is identifying an equivalent expression or solving equations, restrictions on the domain will be a part of the item, either in the question or the answer or both.

Composition of functions:  $f(g(x))$

Variables with subscripts involving exponents:  $v_0^2$   $R_1^2$

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples

October 2008

## ADP Algebra I and Algebra II End-of-Course Exam Notation Information

### Notation on Algebra II End-of-Course Exams – Modules

Correlation coefficient:  $r$

Combinations/Permutations:  ${}_5C_2$   ${}_5P_2$

Matrix elements:  $a_{32}$  for the element in row 3 column 2

Vectors:  $(0, 1)$   $\mathbf{u}$   $\mathbf{AB}$   $\vec{\mathbf{AB}}$

Spreadsheet applications: A1 for column A row 1; \* for multiplication; / for division; ^  
for exponents

# Achieve ADP Algebra II End-of-Course Exam Content Standards with Comments & Examples

October 2008



## ADP Algebra II End-of-Course Exam Expectations of Geometry Knowledge

The following topics are mathematical concepts with which students entering an Algebra II curriculum should be familiar from prior mathematics courses. The high school curriculum or course sequence that a student might follow that leads them to this exam varies by state, district, and sometimes even school. Some curriculum sequences include an integrated series or geometry before or after Algebra II. Regardless of the course sequence followed, the mathematical concepts below are typically considered middle school concepts and taught before the Algebra II, or its equivalent, course(s).

### Prior geometry knowledge/topics where formulas would not necessarily be provided:

- Sum of the interior angles of a triangle equals  $180^\circ$
- Perimeter of polygons
- Area of rectangles
- Area of triangles (not requiring trigonometry)
- Area and circumference of circles
- Surface area of right prisms with rectangular or triangular bases
- Volume of rectangular prisms
- Pythagorean Theorem
- Similar figures

### Use of $\pi$ :

When specified that an exact answer is required, answers should be expressed in terms of  $\pi$ . If not specified, answers may be expressed in terms of  $\pi$ , or 3.14 or  $\frac{22}{7}$  may be used as an approximation for  $\pi$ .