

American Diploma ProjectSM

ALGEBRA II

End-of-Course Exam

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**RELEASED ITEMS
COMMENTARIES**

OCTOBER 2008

*Preparing Today's
High School Students for
Tomorrow's Opportunities*

PEARSON





Created by the nation's governors and business leaders, Achieve, Inc., is a bipartisan non-profit organization that helps states raise academic standards, improve assessments and strengthen accountability to prepare all young people for postsecondary education and training, careers, and citizenship.



Pearson was awarded the contract to develop, deliver, score, and report the ADP Algebra II End-of-Course Exam. Through its Educational Measurement group, Pearson is the largest comprehensive provider of educational assessment products, services and solutions. As a pioneer in educational measurement, Pearson has been a trusted partner in district, state and national assessments for more than 50 years. Pearson helps educators and parents use testing and assessment to promote learning and academic achievement.

Background

The American Diploma Project (ADP) Network includes 33 states (as of September 2008) dedicated to making sure every high school graduate is prepared for college and work. Together, Network members are responsible for educating nearly 80% of all U.S. public school students. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments, and curriculum, and better aligning these expectations with the demands of postsecondary education and work.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam for Algebra II.

In the fall of 2005, nine states—Arkansas, Indiana, Kentucky, Maryland, Massachusetts, New Jersey, Ohio, Pennsylvania and Rhode Island—came together to develop specifications for a common end-of-course exam in Algebra II. These states were planning to require or strongly encourage students to take Algebra II, or its equivalent, in order to better prepare them for college and careers, as Algebra II, or its equivalent, is a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using an end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a higher quality test faster and at lower cost to each state, and to compare their performance and progress with one another.

Five additional states—Arizona, Hawaii, Minnesota, North Carolina, and Washington—have joined the partnership to create the American Diploma Project (ADP) Algebra II End-of-Course Exam, bringing the total number of participating states to fourteen.

In Spring 2008, the exam was administered to nearly 90,000 students across 12 of the consortium states, for the first time as an operational assessment. In future years, the exam will be administered twice each year, at the end of the fall semester and at the end of the spring semester. This document presents a sample of items that appeared either as operational or field-test items in that administration.

The ADP Algebra II End-of-Course Exam serves three main purposes:

1. ***To improve curriculum and instruction—and ensure consistency within and across states.*** The test will help classroom teachers focus on the most important concepts and skills in Algebra II and identify areas where the curriculum needs to be strengthened.
2. ***To help colleges determine if students are ready to do credit-bearing work.*** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to enter and succeed in first-year, credit-bearing mathematics courses. Postsecondary institutions will be able to use the results of the test to tell high school students whether they are ready for college-level work, or if they have content and skill gaps that need to be filled before they enroll in college. This information should help high schools better prepare their students for college, and reduce the need for colleges to provide costly remediation courses.
3. ***To compare performance and progress among the participating states.*** Having agreed on the core content expectations of Algebra II, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators

and the public assess performance, identify areas for improvement, and evaluate the impact of state strategies for improving secondary math achievement.

Algebra II level curriculum: Function modeling and problem solving is the heart of the curriculum at the Algebra II level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables, or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information, and decide whether an exact or approximate answer is appropriate, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

Algebra II level classroom practices: Effective communication using the language of mathematics is essential in a class engaged in content at the Algebra II level. Correct use of mathematical definitions, notation, terminology, syntax, and logic should be required in all work at the Algebra II level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in the *ADP Algebra II End-of-Course Exam Content Standards (Exam Standards)*.

The *Exam Standards* consist of a common set of core standards and seven optional modules (posted at: <http://www.achieve.org/AlgebraIIExamOverview>). The Spring 2008 exam was based upon only the common core standards content.

The Core Algebra II End-of-Course Exam: The core Algebra II End-of-Course Exam covers a range of algebraic topics. Successful students will:

- demonstrate conceptual understanding of the properties and operations of real and complex numbers
- be able to make generalizations through the use of variables resulting in facility with algebraic expressions
- solve single and systems of linear equations and inequalities and be able to use them to represent contextual situations
- be able to demonstrate facility with estimating and verifying solutions of various non-linear equations, making use of technology where appropriate to do so
- demonstrate knowledge of functions and their properties—distinguishing among quadratic, higher degree polynomial, exponential, and piecewise-defined functions—and recognize and solve problems that can be modeled by these functions
- be required to analyze the models, both symbolically and graphically, and determine and effectively represent their solution(s)

There are three types of test items developed that will assess this content: multiple-choice (worth 1 point each), short-answer (worth 2 points each), and extended-response items (worth 4 points each). At least one-third of the student's score will be based on the short-answer and extended-response items. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard, requiring students to make connections and, where appropriate, solve rich contextual problems.

The Exam is comprised of two sessions, with only the second session allowing calculator usage. Although untimed, the exam is designed to take approximately 90 minutes per session.

However, students should be allowed additional time to complete the test, if necessary. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

- Operations and Expressions 15%
- Equations and Inequalities 20%
- Polynomial and Rational Functions 30%
- Exponential Functions 20%
- Functional Operations and Inverses 15%

Algebra II End-of-Course Exam calculator use: The appropriate and effective use of technology is an essential practice in the Algebra II classroom. At the same time, students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra II End-of-Course Exam and should be expected in classrooms where students are working at the Algebra II level. It is therefore important that the Algebra II End-of-Course Exam reflect both practices. For purposes of the Algebra II End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions, and use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities.

Students should use the calculator they are accustomed to and use every day in their classroom work. It should be noted that not all items found on the calculator portion of the exam require the use of a calculator. It is important that students learn to assess for themselves whether or not a calculator would be helpful.

Released Item Commentaries

As stated earlier, these released items are intended to offer insight into the notation, format and expectations of this exam. Of course, no set of items could possibly exemplify all of the available information surrounding an exam such as this one and this set of items should not be considered an exhaustive set of examples. These sample solutions are provided as guidance to teachers and students that will be participating in the ADP Algebra II End-of-Course Exam. Please note that the format of and style of text in the solutions and commentaries does not reflect exactly how items appear in the test books. Some of the released items have been selected because they illustrate a particular type of notation with which students should become familiar; some of the released items attempt to show the intended assessment limit of a particular standard; still others demonstrate the use of diagrams, graphs, or contextual problem situations.

Sample solutions are provided as guidance and should not be considered the only way to solve a problem, nor the only way to reach a particular score point. On the actual Algebra II End-of-Course Exam, the student's choice of solution method might be limited by the placement of the items in the calculator or non-calculator session of the exam. Students should be able to solve test problems in multiple ways, with and without a calculator. The solutions presented in this document are aligned with the session in which an item appears, however, most of the items could have appeared in either session.

For each item, this commentary will include the following:

- item type
- item calculator usage
- benchmark
- item
- the correct answer (multiple choice)
- distractor analyses (multiple choice) to indicate common errors that students could make
- sample solution method
- a sample response at each score point (constructed response)

Item 1

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: E2.b Solve single variable quadratic equations and inequalities over the complex numbers; graph real solution sets on a number line.

Item:

00. The numbers $i\sqrt{3}$ and $-i\sqrt{3}$ are solutions to which of the following equations?

A. $p^2 + \sqrt{3} = 0$

B. $p^2 + 3 = 0$

C. $p^2 + 9 = 0$

D. $p^2 - 9 = 0$

4057MC_2

Correct Answer: B

Explanation of Distractors:

A	Multiplies square roots incorrectly, squaring i but not $\sqrt{3}$.
C	Multiplies square roots incorrectly, squaring 3 instead of $\sqrt{3}$.
D	Multiplies square roots and i incorrectly, squaring 3 instead of $\sqrt{3}$.

Sample Solution:

This problem attempts solving a single variable quadratic equation in reverse. The students are given the solutions and they must identify the equation to which the solutions belong. Solving this type of problem requires the student to recognize that the solutions are roots and can be expressed in factors of the quadratic.

$$(p + i\sqrt{3})(p - i\sqrt{3}) = 0$$

In addition, they must understand the definition of imaginary numbers, $i^2 = -1$.

$$p^2 - ip\sqrt{3} + ip\sqrt{3} - i^2(\sqrt{3})^2 = 0$$

$$p^2 - (-1)(3) = 0$$

$$p^2 + 3 = 0$$

Item 2

Item Type: Multiple Choice

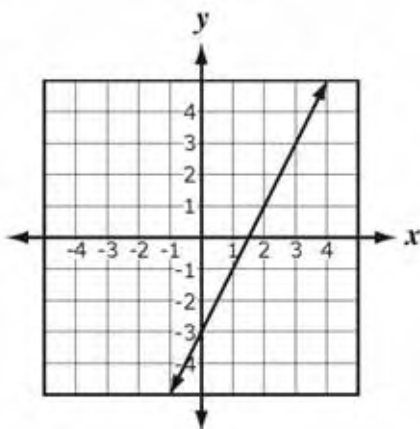
Calculator: NOT permitted

Benchmark: F2.b Determine and graph the inverse relation of a function.

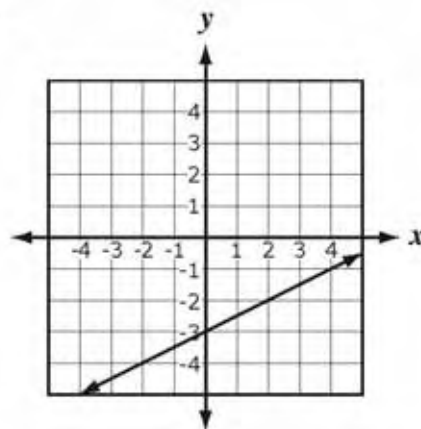
Item:

00. Which graph represents the inverse of $f(x) = 2x - 3$?

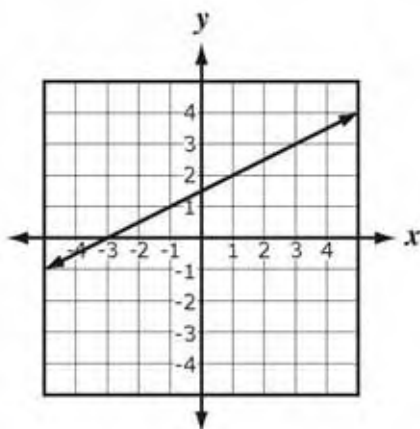
A.



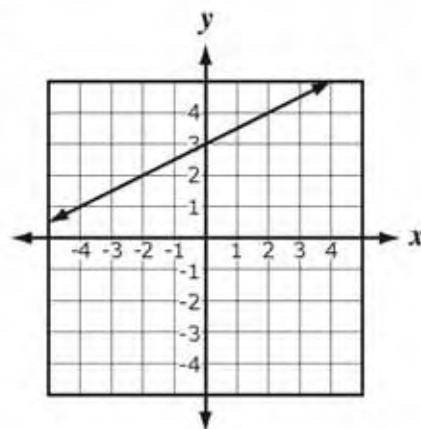
B.



C.



D.



8315MC_3

Correct Answer: C

Explanation of Distractors:

A	Graphed $f(x)$.
B	Found the reciprocal of the slope.
D	Did not divide 3 by 2 when finding the inverse.

Sample Solution:

One method students can use to solve this item is to begin by finding the inverse function $f^{-1}(x)$ by replacing x with y and replacing $f(x)$ with x , then solving for y .

$$\begin{aligned}x &= 2y - 3 \\x + 3 &= 2y \\y &= \frac{1}{2}x + \frac{3}{2} \\f^{-1}(x) &= \frac{1}{2}x + \frac{3}{2}\end{aligned}$$

From the inverse function, the correct answer is the graph with a y-intercept of $\frac{3}{2}$ and a slope of $\frac{1}{2}$.

Another method that students can use to solve this problem is to begin by determining some of the points that are generated by the function $f(x)$.

x	$f(x)$
0	-3
1	-1
2	1

Then students can swap the coordinates of each ordered pair to determine ordered pairs of $f^{-1}(x)$.

x	$f^{-1}(x)$
-3	0
-1	1
1	2

The graph for answer choice C is the graph that contains the above points.

Item 3

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: O3.c Apply the laws of exponents to algebraic expressions, including those involving rational and negative exponents, to order and rewrite them in alternative forms.

Item:

00. Given r , s , and t are positive integers, which of the following is equivalent to $r^{\left(-\frac{s}{t}\right)}$?

A. $-\sqrt[t]{r^s}$

B. $\sqrt[s]{r^t}$

C. $-\frac{1}{\sqrt[s]{r^t}}$

D. $\frac{1}{\sqrt[t]{r^s}}$

6101MC_4

Correct Answer: D

Explanation of Distractors:

A	Raising a base to a negative exponent negates the expression. $-r^{\frac{s}{t}} = -\sqrt[t]{r^s}$
B	Raising a base to a negative exponent results in the reciprocal of the exponent. $r^{\frac{t}{s}} = \sqrt[s]{r^t}$
C	Raising a base to a negative exponent results in the reciprocal of the base and the exponent and negates the expression. $-\frac{1}{r^{\frac{t}{s}}} = -\frac{1}{\sqrt[s]{r^t}}$

Sample Solution:

Students must understand the definition of rational and negative exponents to identify the correct conversion, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where a is a positive real number, m and n are integers, and $n > 0$, with $a^{\frac{m}{n}} = \frac{1}{\frac{m}{a^n}}$ when $\frac{m}{n} < 0$.

$$r^{\left(-\frac{s}{t}\right)} = \left(\frac{1}{r^{\frac{s}{t}}}\right) = \frac{1}{\sqrt[t]{r^s}}$$

Item 4

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: E1.a Solve equations and inequalities involving the absolute value of a linear expression.

Item:

00. How many solutions does the equation $|-1 + 3x| - 5x = 9$ have?

- A. 0
- B. 1
- C. 2
- D. 3

6138MC_2

Correct Answer: B

Explanation of Distractors:

A	Excludes both the extraneous and correct solutions.
C	Includes extraneous solution.
D	The left side of the equation has three terms.

Sample Solution:

To determine the number of solutions, students can start by solving for the absolute value part of the expression and then break it into two separate equations to solve.

$$\begin{aligned}|-1 + 3x| - 5x &= 9 \\|-1 + 3x| &= 5x + 9 \\-1 + 3x &= 5x + 9 \quad \text{or} \quad -1 + 3x = -(5x + 9) \\-2x &= 10 & -1 + 3x &= -5x - 9 \\x &= -5 & 8x &= -8 \\ & & x &= -1\end{aligned}$$

The numbers -5 and -1 are possible solutions, but because there was a variable outside the absolute value sign, students must check if these solutions are valid.

$$|-1 + 3(-5)| - 5(-5) \stackrel{?}{=} 9$$

$$|-1 - 15| + 25 \stackrel{?}{=} 9$$

$$|-16| + 25 \stackrel{?}{=} 9$$

$$16 + 25 \stackrel{?}{=} 9$$

$$41 \neq 9$$

$$|-1 + 3(-1)| - 5(-1) \stackrel{?}{=} 9$$

$$|-1 - 3| + 5 \stackrel{?}{=} 9$$

$$|-4| + 5 \stackrel{?}{=} 9$$

$$4 + 5 \stackrel{?}{=} 9$$

$$9 = 9$$

Students must recognize -5 is an extraneous solution, leaving -1 as the only solution.

Item 5

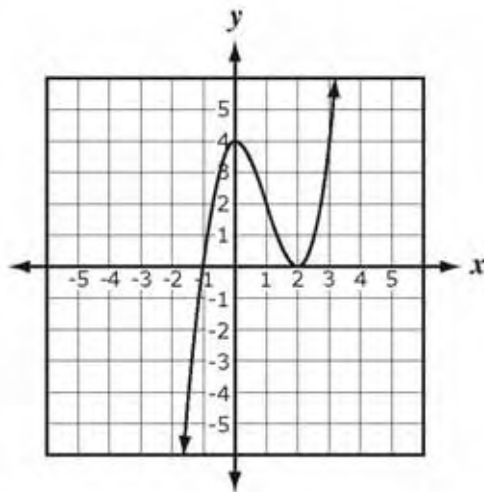
Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: P2.b Determine key characteristics of polynomial functions and their graphs.

Item:

00. The graph of a polynomial function $f(x)$ is shown below.



What could be the factorization of the polynomial $f(x)$?

- A. $(x - 1)(x + 4)(x + 2)$
- B. $(x + 1)(x - 4)(x - 2)$
- C. $(x + 1)(x - 2)(x - 2)$
- D. $(x)(x + 1)(x - 2)$

2009MC_3

Correct Answer: C

Explanation of Distractors:

A	Uses all of the x -intercepts and the y -intercept with the wrong signs.
B	Uses all of the x -intercepts and the y -intercept.
D	Uses the x -intercepts and the x -coordinate for the y -intercept.

Sample Solution:

Students should be familiar with the relationships between a higher-order polynomial function and its graph. This includes the relationship of the roots of a function and their multiplicity to the x -intercepts of the graph. The x -intercepts of -1 and 2 in the given graph are used to determine the factors $(x + 1)$ and $(x - 2)$ of the polynomial $f(x)$. Since the graph changes direction at the x -intercept of 2 , the polynomial has a double root at 2 .

$$f(x) = (x + 1)(x - 2)(x - 2)$$

The correct answer represents the function with the minimum possible degree (third degree) since it is possible for the graph to have more intercepts outside the range of the graph or changes in direction that are too small to see inside the range of the graph.

Item 6

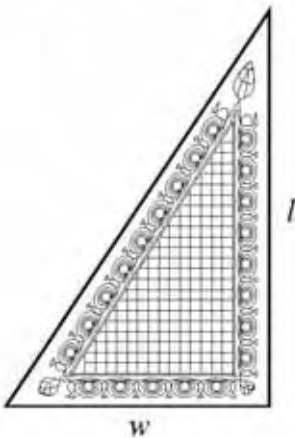
Item Type: Short Answer

Calculator: NOT permitted

Benchmark: E2.a Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.

Item:

00. The area of a right triangular rug is 12 square feet. The length (l) of the rug is 2 feet more than the width (w) of the rug.



Find the width of the right triangular rug. Show or explain your work.

15825A_S

Correct Answer: 4 feet

Scoring Rubric:

Score	Description
2	Student earns 2 points.
1	Student earns 1 point.
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

2 points

- 1 point for a correct strategy

$$\left[\begin{array}{l} \frac{1}{2}(w+2)w = 12 \\ w^2 + 2w = 24 \\ w^2 + 2w - 24 = 0 \\ (w+6)(w-4) = 0 \\ \text{or equivalent} \end{array} \right]$$

- 1 point for a correct answer [4 and evidence of excluding -6] or a correct answer based on an incorrect quadratic equation

Sample Solutions:

In this sample solution, the student earned a score of 2. A correct strategy was used in writing a correct quadratic equation. Also it was solved correctly and the negative answer was excluded. This is shown here by the circling of what the student intended to be his or her answer to the problem.

$A = 12$
 $l = w + 2$
 $12 = \frac{1}{2}w(w+2)$
 $24 = w^2 + 2w$
 $w^2 + 2w - 24 = 0$
 $(w+6)(w-4) = 0$
 $w+6 = 0$ or $w-4 = 0$
 $w = -6$ $w = 4$
 width = 4 ft

In this sample of a response that earned a score of 1, the student began with the wrong equation as he or she did not multiply by $\frac{1}{2}$ when finding the area, so the point for strategy was not awarded. However, since the equation was quadratic, was solved correctly, and the negative answer was excluded, as evidenced by the positive answer being circled, the student earned the point for having a correct answer based on an incorrect equation.

$$\begin{aligned}
 \text{width} &= w \\
 \text{length} &= w + 2 \\
 w(w + 2) &= 12 \\
 w^2 + 2w &= 12 \\
 w^2 + 2w - 12 &= 0 \\
 w &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-12)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 48}}{2} = \frac{-2 \pm \sqrt{52}}{2} \\
 &= \frac{-2 \pm 2\sqrt{13}}{2} = -1 \pm \sqrt{13} \\
 &= -1 + \sqrt{13} \text{ feet}
 \end{aligned}$$

The following is a sample of a response that earned a score of 0. In this response the student did not get the point for a correct strategy because the equation that he or she wrote is not a correct method to find area. The student did solve his or her equation correctly, but did not get a point for a correct answer based on an incorrect equation because the equation was not quadratic.

$$\begin{aligned}
 \text{width} &= w \\
 \text{length} &= w + 2 \\
 w + (w + 2) &= 12 \\
 2w + 2 &= 12 \\
 2w &= 10 \\
 w &= 5
 \end{aligned}$$

Item 7

Item Type: Short Answer

Calculator: NOT permitted

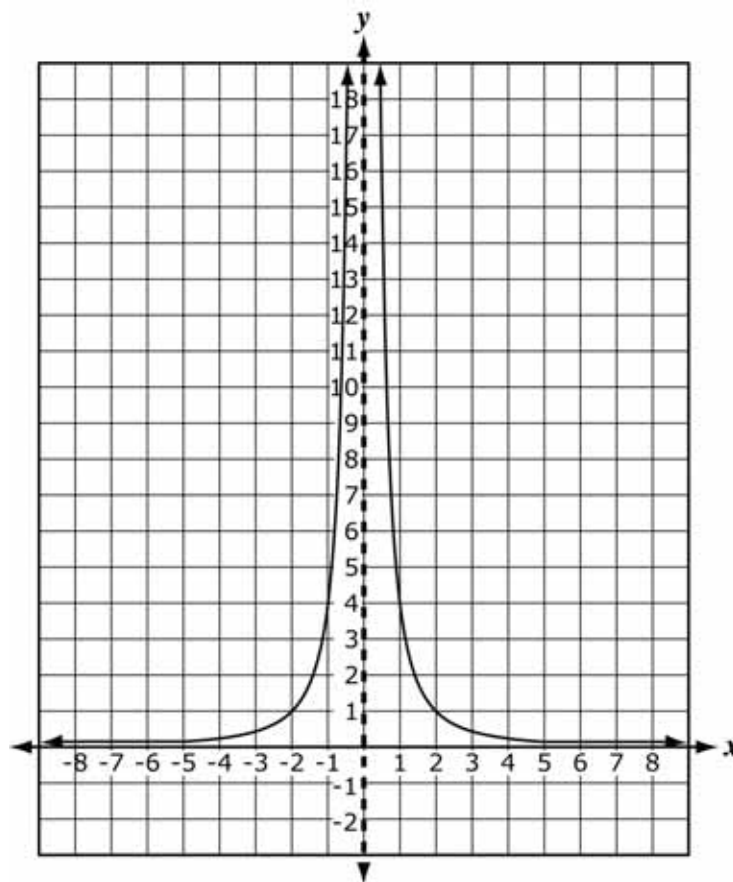
Benchmark: P2.e Represent simple rational functions using tables, graphs, verbal statements, and equations. Translate among these representations.

Item:

00. Graph $f(x) = \frac{4}{x^2}$. Label the axes and scales used to construct the graph.

1401SA_5

Correct Answer:



Scoring Rubric:

Score	Description
2	Student earns 3 points.
1	Student earns 1 or 2 points.

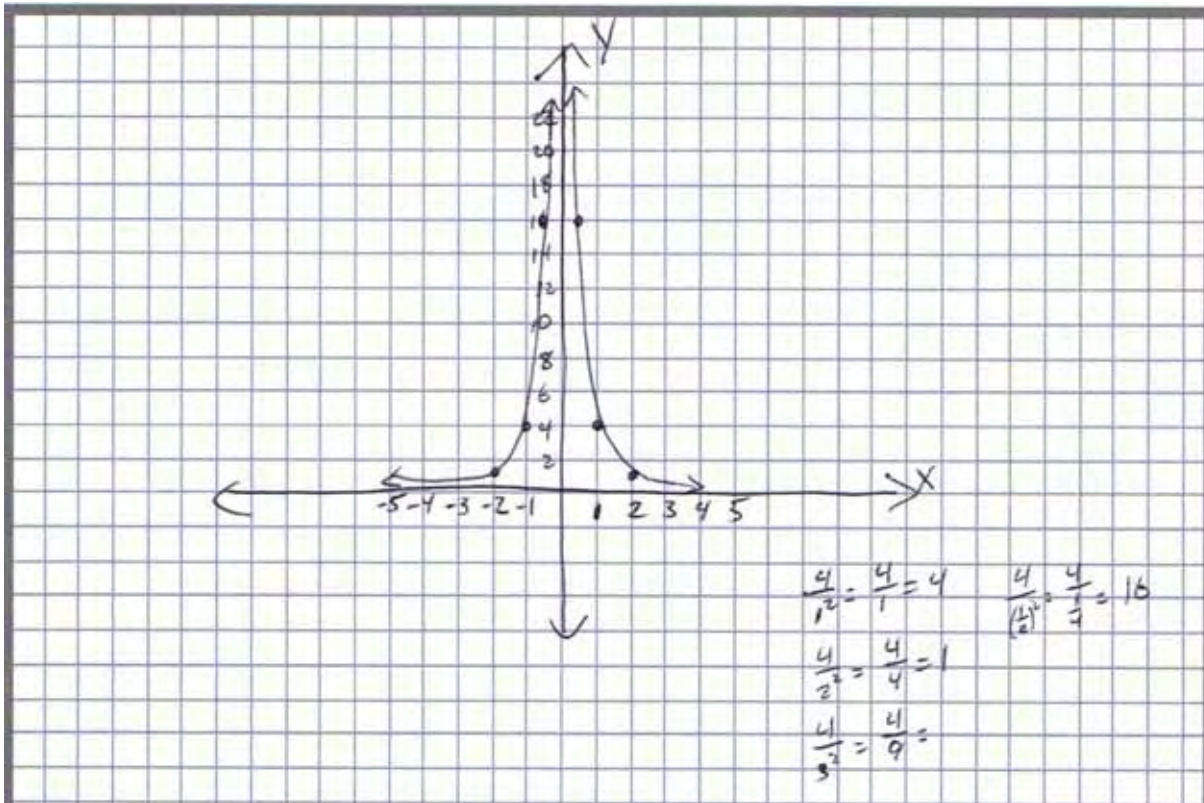
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

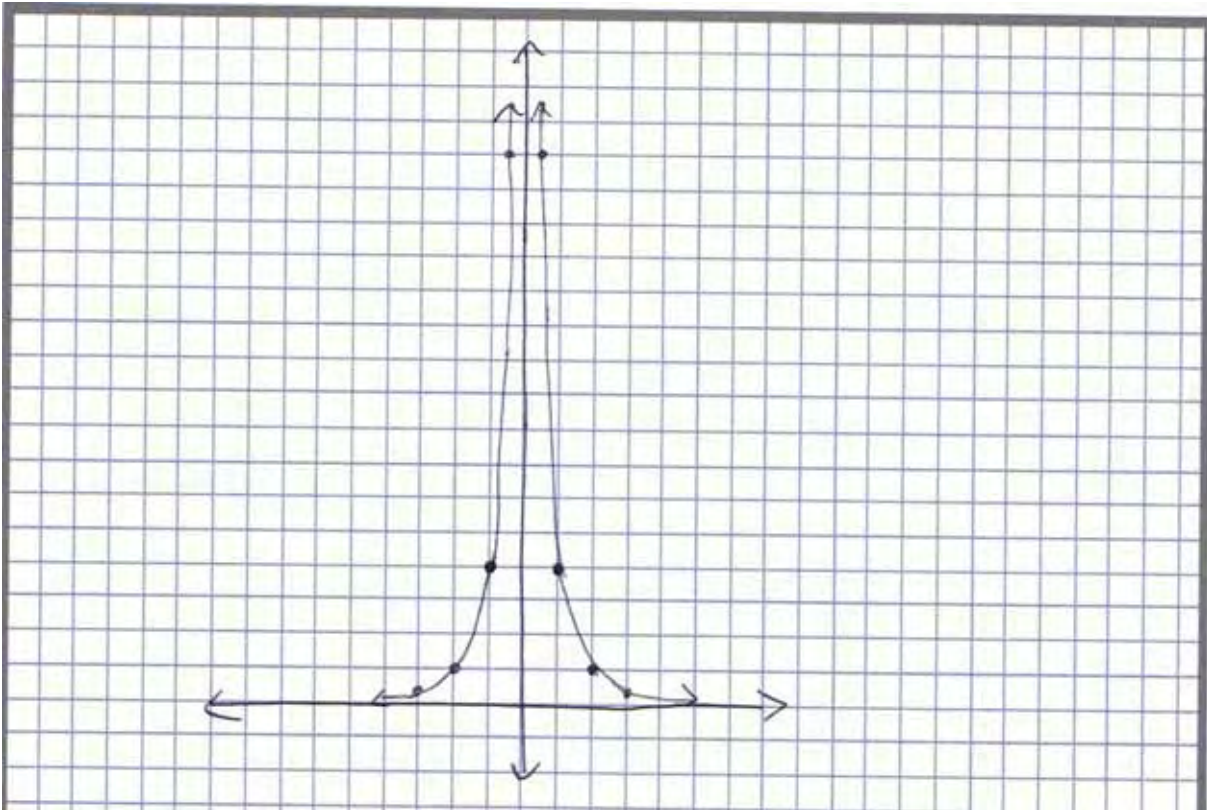
- 1 point for a related graph with appropriate labels for the axes and scales
- 1 point for the correct shape of the graph [symmetrical about the vertical asymptote $x = 0$ and has a horizontal asymptote of $y = 0$]
- 1 point for the correct graph [approximately passing through $(-2, 1)$, $(-1, 4)$, $(-0.5, 16)$, $(0.5, 16)$, $(1, 4)$, $(2, 1)$ or equivalent]

Sample Solutions:

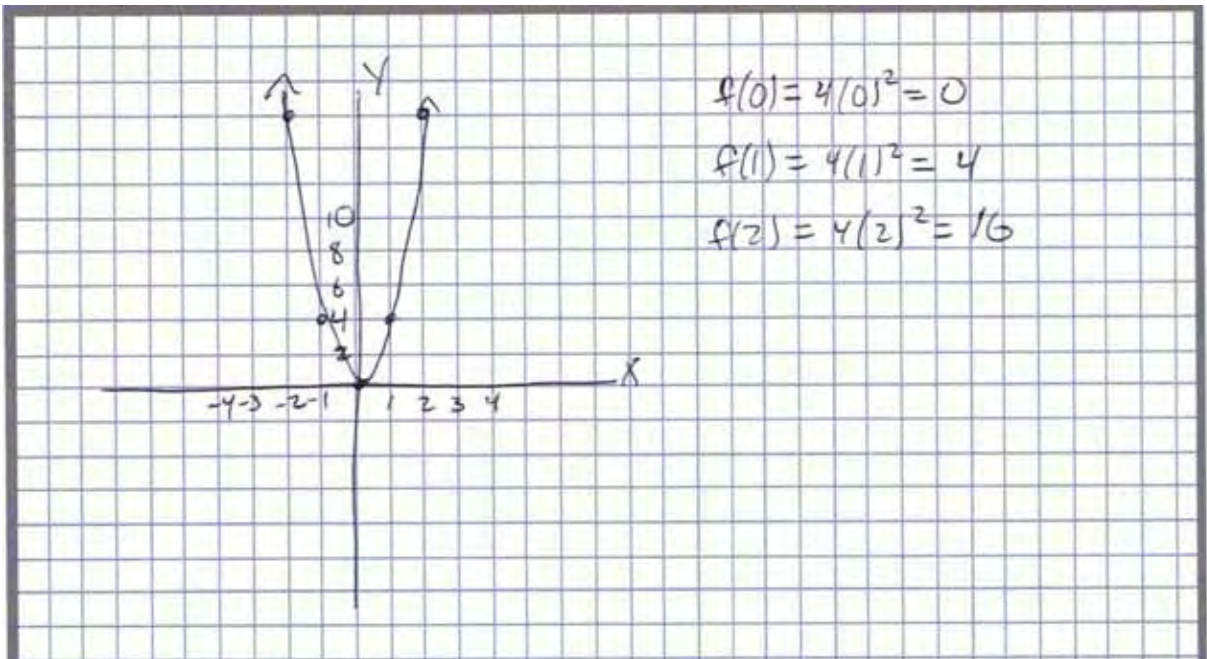
This is a sample of a response that earned a score of 2. The axes and scales were labeled and the shape of the graph is correct as it passes through the correct points. The work that is shown below the graph is not necessary to earn the point as the item does not ask for work and it is not listed in the rubric.



The response on the following page is a sample of a response that earned a score of 1. The response did not earn the first point for being a related graph because, although it was a related graph, the axes and scales are not labeled. It did earn the other two points, for having the correct approximate shape and for passing through the correct points. Since there are not any scales indicated in the response, it was scored under the assumption that each grid line represents 1 unit.



This final sample shows a response which received a score of 0. While the axes are labeled and include appropriate scales, the graph shown is not a related graph. For the purposes of earning the point for a related graph, the graph would have to show a rational function. This graph shows a quadratic function instead.



Item 8

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: X1.a Determine key characteristics of exponential functions and their graphs.

Item:

00. Consider the table below.

Function Characteristics

Characteristic	$f(x)$
Domain	all real numbers
Range	$y > 1$
Intercept	(0,2)
Asymptote	$y = 1$
As x approaches $+\infty$	y approaches $+\infty$
As x approaches $-\infty$	y approaches 1

Which function has the characteristics described in this table?

- A. $f(x) = 2^x$
- B. $f(x) = -2^x$
- C. $f(x) = 3^x + 1$
- D. $f(x) = -3^x + 1$

2262MC_3

Correct Answer: C

Explanation of Distractors:

A	Chooses a growth function, without a vertical shift.
B	Chooses the negative of a growth function.
D	Chooses the negative of a growth function with a vertical shift.

Sample Solution:

In order to correctly choose the function with given characteristics, students must understand the relationships between an exponential function and its graph. For instance, the given range of $y > 1$ indicates that, in the exponential form $f(x) = a \cdot b^x$, a is positive. The statements "as x approaches $+\infty$, y approaches $+\infty$ " and "as x approaches $-\infty$, y approaches 1" indicate that b in the exponential form is positive, and that the exponential function is a growth function. Since the y -intercept is at $(0, 2)$ instead of $(0, 1)$, there is a vertical shift of 1, which is indicated by adding 1 to the exponential form.

$$f(x) = 3^x + 1$$

Item 9

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: F3.a Determine key characteristics of absolute value, step, and other piecewise-defined functions.

Item:

00. What is the slope of the graph of $f(x) = -\frac{1}{2}|x-1|+4$ when $x < 1$?

- A. -2
- B. $-\frac{1}{2}$
- C. $\frac{1}{2}$
- D. 2

8279MC_3

Correct Answer: C

Explanation of Distractors:

A	Multiplied $-\frac{1}{2}$ and 4.
B	Used the coefficient of the absolute value part of the function as the slope.
D	Found the reciprocal of the slope.

Sample Solution:

To find the slope of $f(x)$ when $x < 1$, students could begin by writing the function as a piecewise function. The vertex of the function is at $(1, 4)$ from the form $f(x) = a|x-h|+k$.

$$f(x) = \begin{cases} -\frac{1}{2}(-(x-1)) + 4, & x < 1 \\ -\frac{1}{2}(x-1) + 4, & x \geq 1 \end{cases}$$

Then, students can simplify the portion of the function that applies when $x < 1$.

$$f(x) = -\frac{1}{2}(-(x-1)) + 4 = \frac{1}{2}(x-1) + 4 = \frac{1}{2}x + \frac{7}{2}$$

The slope is $\frac{1}{2}$.

Item 10

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: E1.c Solve systems of linear inequalities in two variables and graph the solution set.

Item:

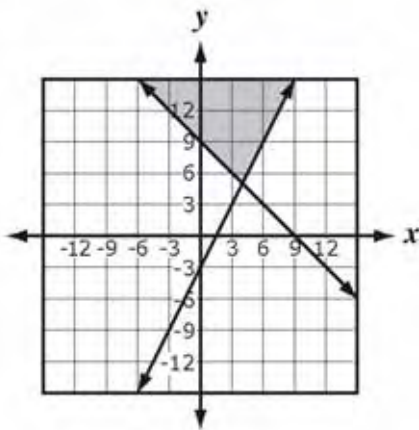
00. Consider the system of inequalities below.

$$y \geq 2x - 3$$

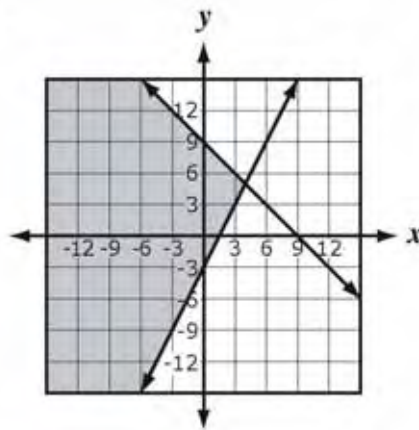
$$y \leq -x + 9$$

Which graph represents this system?

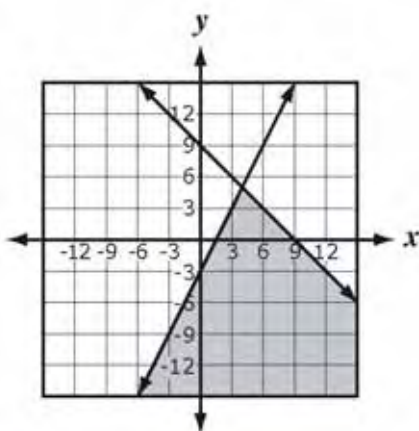
A.



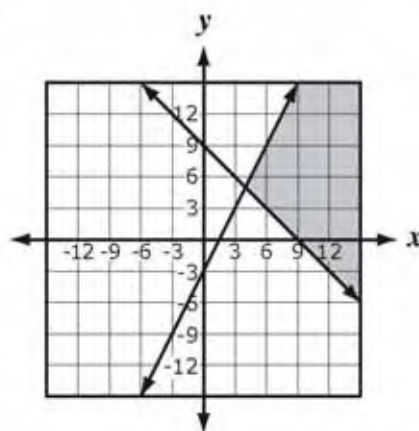
B.



C.



D.



9101MC_2

Correct Answer: B

Explanation of Distractors:

A	Solution of $y \geq 2x - 3$ and $y \geq -x + 9$.
C	Solution of $y \leq 2x - 3$ and $y \leq -x + 9$.
D	Solution of $y \leq 2x - 3$ and $y \geq -x + 9$.

Sample Solution:

Both inequalities are in slope-intercept form, so students can distinguish between the two lines using either slopes, as one is negative and the other is positive, or by identifying the y-intercepts of each line. Whichever method students use, they should recognize that $y \geq 2x - 3$ corresponds to the line that passes through $(0, -3)$ and has a slope of 2, and $y \leq -x + 9$ corresponds to the line that passes through $(0, 9)$ and has a slope of -1 .

Since y is greater than or equal to the expression of x in $y \geq 2x - 3$, the shading relative to its line should be above the line. Likewise, the shading relative to $y \leq -x + 9$ should be below its line since y is less than or equal to the expression of x in that inequality. Answer choice B is the one graph of the four shown that meets these criteria.

Item 11

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: F3.c Recognize, express, and solve problems that can be modeled using absolute value, step, and other piecewise defined functions. Interpret their solutions in terms of the context.

Item:

00. A website uses the following table to calculate the shipping charge (S) of an order based on the order's total cost (c).

Shipping Charges

Order Total Cost (c)	Shipping Charges (S)
Up to \$14.99	\$5.00
\$15.00–24.99	\$7.00
\$25.00–34.99	\$9.00
\$35.00–44.99	\$11.00
\$45.00–54.99	\$13.00

Which greatest integer function can be used to compute the accurate shipping charges based on the information in this table?

A. $S(c) = 5 + 2[c - 15]$

B. $S(c) = 5 + 2\left[\frac{c}{15}\right]$

C. $S(c) = \begin{cases} 5, & 0 < c < 15 \\ 5 + 2\left[\frac{c-5}{10}\right], & c \geq 15 \end{cases}$

D. $S(c) = \begin{cases} 5, & 0 < c < 15 \\ 5 + 2\left[\frac{c-15}{10}\right], & c \geq 15 \end{cases}$

2351MC_3

Correct Answer: C

Explanation of Distractors:

A	This function does not account for the shipping charges increasing by \$2 for every \$10 after the first \$14.99 in order total cost.
B	With this function, the second \$10 increase after the first \$14.99 in order total cost would cause the shipping charges to be less than the given \$9. For example: $S(25) = 5 + 2 \left[\frac{25}{15} \right] = 5 + 2(1) = 7$
D	With this piece-wise function, the first \$10 increase after the first \$14.99 in order total cost would cause the shipping charges to be less than the given \$5. For example: $S(15) = 5 + 2 \left[\frac{0}{10} \right] = 5 + 0 = 5$

Sample Solution:

Students must understand that a greatest integer function returns the value of the greatest integer less than or equal to the number given in square brackets. In other words, the greatest integer function rounds down to the next smallest integer from the number in the brackets, unless the number in brackets is itself an integer in which case the number remains unchanged. For example, if $f(x) = [x]$, then $f(2.25) = [2.25] = 2$, $f(1.9) = [1.9] = 1$, $f(3) = [3] = 3$, and $f(-4.3) = [-4.3] = -5$.

The table of shipping charges requires students to recognize that not only are the charges constant up to \$14.99 of order total cost, but that they increase at a constant rate of \$2 for each additional \$10 after the first \$14.99 of order total cost.

The first part of the function is \$5 for values of c between \$0 and \$15. This is because the cost remains constant for the amounts up to \$14.99.

To determine the second part of the function for values of c that are \$15 or greater, students need to calculate the number of \$10 price increases that is associated with the \$2 increase in shipping charges. This means that for $c \geq 15$, increasing whole numbers must be generated for each new \$10 range starting at \$15. For example, 1 should be multiplied by 2 and then added to 5 for any cost from \$15 to \$24.99. The student must determine the greatest integer function that yields a value of 1 to 1.99 inside the brackets to generate a 1 that can be multiplied by 2. The only function that yields the correct whole numbers to be multiplied by 2 for $c \geq 15$ is C.

$$S(c) = \begin{cases} 5, & 0 < c < 15 \\ 5 + 2 \left[\frac{c - 5}{10} \right], & c \geq 15 \end{cases}$$

Item 12

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: E2.c Use the discriminant, $D = b^2 - 4ac$, to determine the nature of the solutions of the equation $ax^2 + bx + c = 0$.

Item:

00. A quadratic equation in the form $ax^2 + bx + c = 0$, where a , b , and c are non-zero real numbers, has a negative discriminant. What are the solutions to this equation if m and n are non-zero real numbers?

- A. $x = m$ and $x = n$
- B. $x = m + ni$ and $x = m - ni$
- C. $x = m + n$ and $x = m - n$
- D. $x = m + ni$ and $x = -m - ni$

6186MC_2

Correct Answer: B

Explanation of Distractors:

A	Found possible solutions for an equation with a positive discriminant.
C	Omitted the i from the solutions.
D	Chose the complex solutions that are opposites, not conjugates.

Sample Solution:

Since the discriminant is negative, students need to realize that the quadratic equation has no real solutions, but instead has two complex solutions. When a , b , and c are all non-zero real numbers in the equation $ax^2 + bx + c = 0$, the complex solutions are always conjugates of each other: $x = m + ni$ and $x = m - ni$.

Item 13

Item Type: Multiple Choice

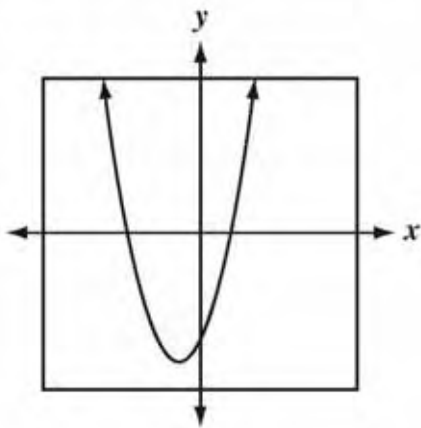
Calculator: NOT permitted

Benchmark: P1.a Determine key characteristics of quadratic functions and their graphs.

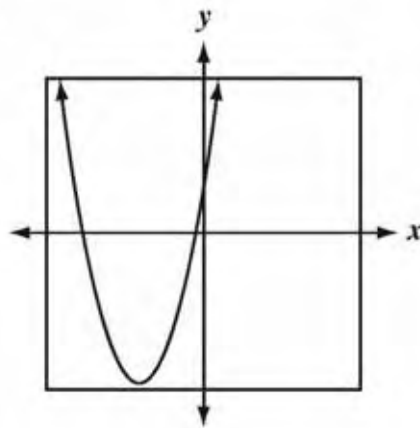
Item:

00. Which graph could represent the function $f(x) = -ax^2 + bx + c$, where a , b , and c are positive integers?

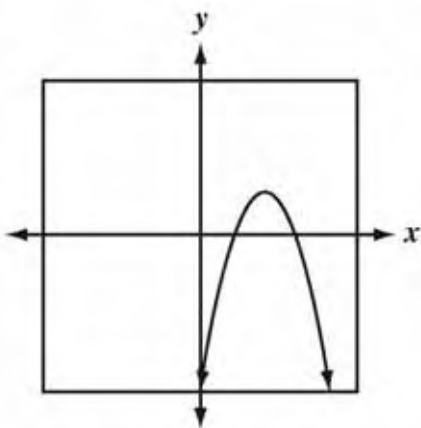
A.



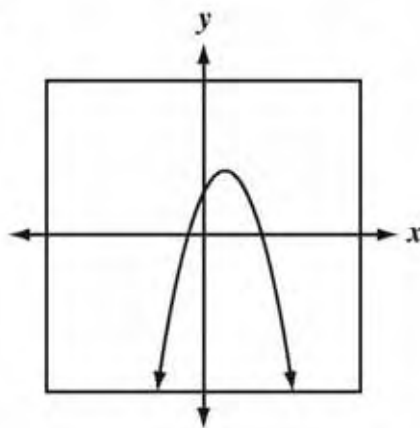
B.



C.



D.



6151MC_4

Correct Answer: D

Explanation of Distractors:

A	The graph shown has a positive value for a and a negative value of c .
B	Since c is positive, chose a parabola opening upward with a positive y -intercept.
C	The coefficient, a , is negative, so the parabola opens downward.

Sample Solution:

Since the coefficient, a , of the squared term of the quadratic function is negative, students can conclude that the graph of $f(x)$ is a parabola opening downward. Since the constant term, c , is positive, students can conclude that the y -intercept of the parabola is positive. The graph in answer choice D is the only one of the four that meets these criteria.

Item 14

Item Type: Short Answer

Calculator: NOT permitted

Benchmark: O3.d Perform operations on polynomial expressions.

Item:

00. A cube that measures 4 inches on each side is increased by m inches on each side.

Part A Determine the expression for the volume of the new cube in terms of its side.

Part B Expand the expression for the volume of the new cube in terms of its side. Show or explain your work.

16985A_5

Correct Answer:

Part A: $(4 + m)^3$

Part B: $64 + 48m + 12m^2 + m^3$

Scoring Rubric:

Score	Description
2	Student earns 3 points.
1	Student earns 1 or 2 points.
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

Part A: 1 point

- 1 point for a correct answer [$(4 + m)^3$ or equivalent]

Part B: 2 point

- 1 point for a correct strategy [Write $(4 + m)(4 + m)(4 + m)$ or $(16 + 8m + m^2)(4 + m)$ or Pascal's Triangle or equivalent]
- 1 point for a correct answer [$64 + 48m + 12m^2 + m^3$ or equivalent] or a correct answer based on an incorrect answer in Part A

Sample Solution:

The sample response earned a score of 2 by receiving all three of the points listed in the scoring notes. For Part A, the student gave a correct expression, and for Part B the student shows a correct strategy, multiplying $(4 + m)(4 + m)(4 + m)$, and gave a correct answer.

Part A $(m+4)^3$

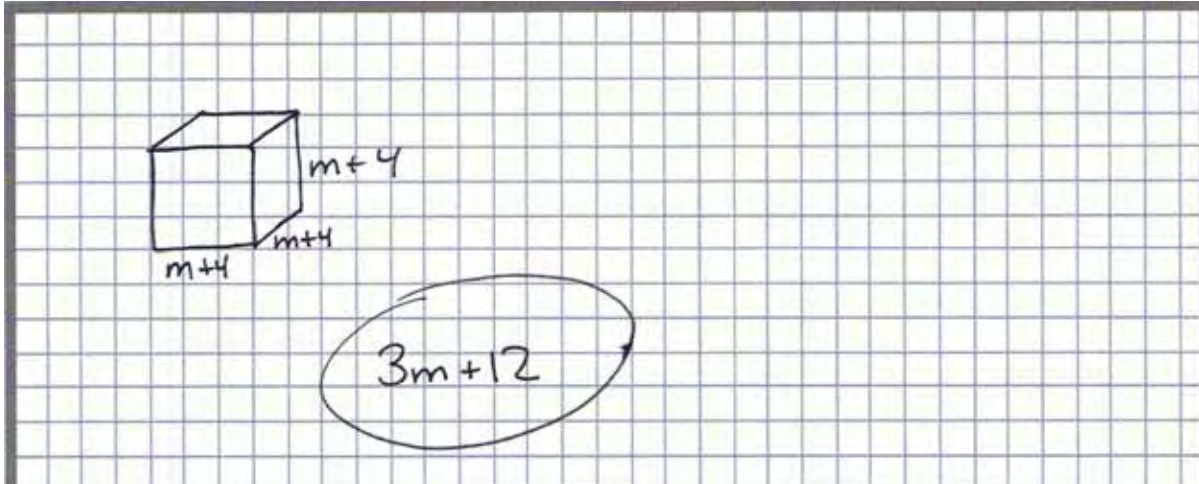
Part B $(m+4)^3 = (m+4)(m+4)(m+4)$
 $= (m^2 + 4m + 4m + 16)(m+4)$
 $= (m^2 + 8m + 16)(m+4)$
 $= m^3 + 4m^2 + 8m^2 + 32m + 16m + 64$
 $= m^3 + 12m^2 + 48m + 64$

This sample response earned a score of 1. It receives the point from Part A for a correct answer. Part B did not earn either of its points though. The strategy used to cube $m + 4$ was incorrect and the answer was incorrect.

Part A $(m+4)^3$

Part B $(m+4)^3 = m^3 + 4^3 = m^3 + 64$

This final sample response earned a score of 0. While the student drew a picture of the situation described in the item, no expression for the volume was written. For the answer to Part B, the student did not show a strategy to find the volume, but appears to have multiplied the side length by 3 instead of cubing it. Without an expression for Part A and explanation or work for Part B, it cannot be determined exactly what procedure was followed.



Item 15

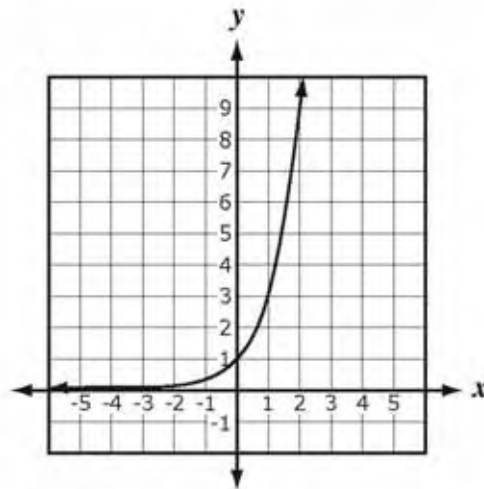
Item Type: Multiple Choice

Calculator: Permitted

Benchmark: X1.c Describe the effect that changes in a parameter of an exponential function have on the shape and position of its graph.

Item:

00. The graph of $f(x) = 3^x$ is shown below.



The graph of which of the following functions intersects the x -axis?

- A. $f(x) = -3^x$
- B. $f(x) = 3^{-x}$
- C. $f(x) = 3^{x-4}$
- D. $f(x) = 3^x - 4$

5053MC_4

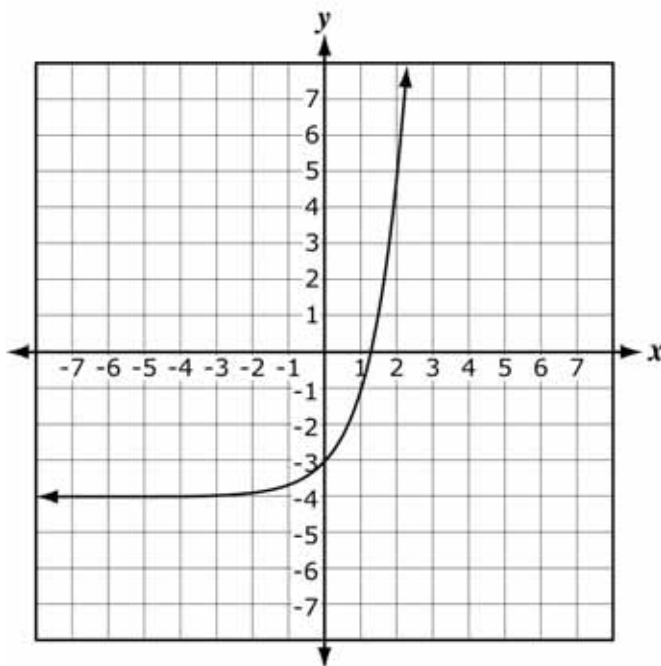
Correct Answer: D

Explanation of Distractors:

A	Reflects the graph over the x -axis, but there are no x -intercepts.
B	Reflects the graph over the y -axis, but there are no x -intercepts.
C	Shifts the graph to the right 4 units, but there are no x -intercepts.

Sample Solution:

To determine which function represents a graph that intersects the x -axis, students should use an understanding about how the changes in each parameter of an exponential function in the form $f(x) = ab^x + c$ affects its graph. If c is negative, then the graph will shift down and cross the x -axis between $x = 1$ and $x = 2$.



Item 16

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: E2.e Rewrite nonlinear equations and inequalities to express them in multiple forms in order to facilitate finding a solution set or to extract information about the relationships or graphs indicated.

Item:

00. The formula below can be used to determine the period (T) of a pendulum in seconds, given the pendulum's length (L) in meters.

$$T = 2\pi\sqrt{\frac{L}{9.8}}$$

Which expression can be used to determine the length (L) of a pendulum given its period (T)?

A. $L = 2\pi\frac{T^2}{9.8}$

B. $L = 4\pi^2\frac{T^2}{9.8}$

C. $L = 9.8\frac{T^2}{2\pi}$

D. $L = 9.8\frac{T^2}{4\pi^2}$

3152MC_4

Correct Answer: D

Explanation of Distractors:

A	Squares both sides of the equation, multiplies by 2π , then divides by 9.8.
B	Multiplies by 2π , squares both sides of the equation, then divides by 9.8.
C	Squares both sides of the equation, divides by 2π , then multiplies by 9.8.

Sample Solution:

The problem explains the formula $T = 2\pi\sqrt{\frac{L}{9.8}}$ to demonstrate the importance of being able to manipulate literal equations to solve for the information that is needed, which, in this case, is the length of a pendulum. There are several different approaches in the order of operations to use in solving for L , but dividing by 2π first then squaring both sides avoids some common errors.

$$\begin{aligned}T &= 2\pi\sqrt{\frac{L}{9.8}} \\ \frac{T}{2\pi} &= \sqrt{\frac{L}{9.8}} \\ \frac{T^2}{4\pi^2} &= \frac{L}{9.8} \\ L &= 9.8 \frac{T^2}{4\pi^2}\end{aligned}$$

Item 17

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: P1.d Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.

Item:

00. At an amusement park Lisa rides The Plunger and The Skydiver. These rides take people up to a certain height and then let them drop to experience free-fall. As Lisa starts her drop for either ride, the function $h(t) = -16t^2 + h_0$ describes her height in feet as a function of time in seconds, where h_0 is her initial height. She reaches the bottom of The Plunger's 144-foot drop 1.5 seconds faster than she reaches the bottom of The Skydiver's drop. What is the height of The Skydiver's drop in feet?

- A. 180
- B. 216
- C. 324
- D. 468

6205MC_3

Correct Answer: C

Explanation of Distractors:

A	Found the difference in the heights of the two rides.
B	Multiplied 144 by 1.5.
D	Found the correct time for The Plunger, but added $16(4.5)^2$ to 144.

Sample Solution:

To solve this problem, students can begin by determining the time it takes for Lisa to drop 144 feet on The Plunger. To do this, students can substitute known values into the function

$h(t) = -16t^2 + h_0$. The initial height (h_0) is 144 and since the student is trying to determine the time to drop to the bottom, the value of $h(t)$ is 0.

$$0 = -16t^2 + 144$$

$$16t^2 = 144$$

$$t^2 = 9$$

$$t = \pm 3$$

In the context of the problem, only the positive time makes sense, so it takes 3 seconds to drop 144 feet on The Plunger. Students can then determine that the time it takes to drop on The Skydiver is 4.5 seconds, which is 1.5 seconds longer than the time for The Plunger, and then substitute 4.5 for t and 0 for $h(t)$ into the function, solving for h_0 .

$$0 = -16(4.5)^2 + h_0$$

$$0 = -324 + h_0$$

$$h_0 = 324$$

So, the height of The Skydiver is 324 feet.

Another method students could use to solve the problem would be to use a graphing calculator to find the time it takes for The Plunger to drop 144 feet. To do this the student would graph the function $y = -16x^2 + 144$ and use the Calculate zero function to determine that the time it takes for The Plunger to drop is 3 seconds. The student could then find the time for The Skydiver by adding 1.5 to 3 and then substitute known values into the function as in the previous method.

Item 18

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: F1.b Determine the composition of two functions, including any necessary restrictions on the domain.

Item:

00. Consider the functions below.

$$k(x) = \frac{1}{\sqrt{x-6}}$$
$$m(x) = 3x$$

What is the domain of $k(m(x))$ over the set of real numbers?

- A. $x > 0$
- B. $x > 2$
- C. $x > 6$
- D. all real numbers

2158MC_2

Correct Answer: B

Explanation of Distractors:

A	Chooses the restrictions for the denominator of the fraction $\frac{1}{x}$.
C	Chooses the restrictions for the denominator of $k(x)$.
D	Chooses the restrictions for $m(x)$, which has none.

Sample Solution:

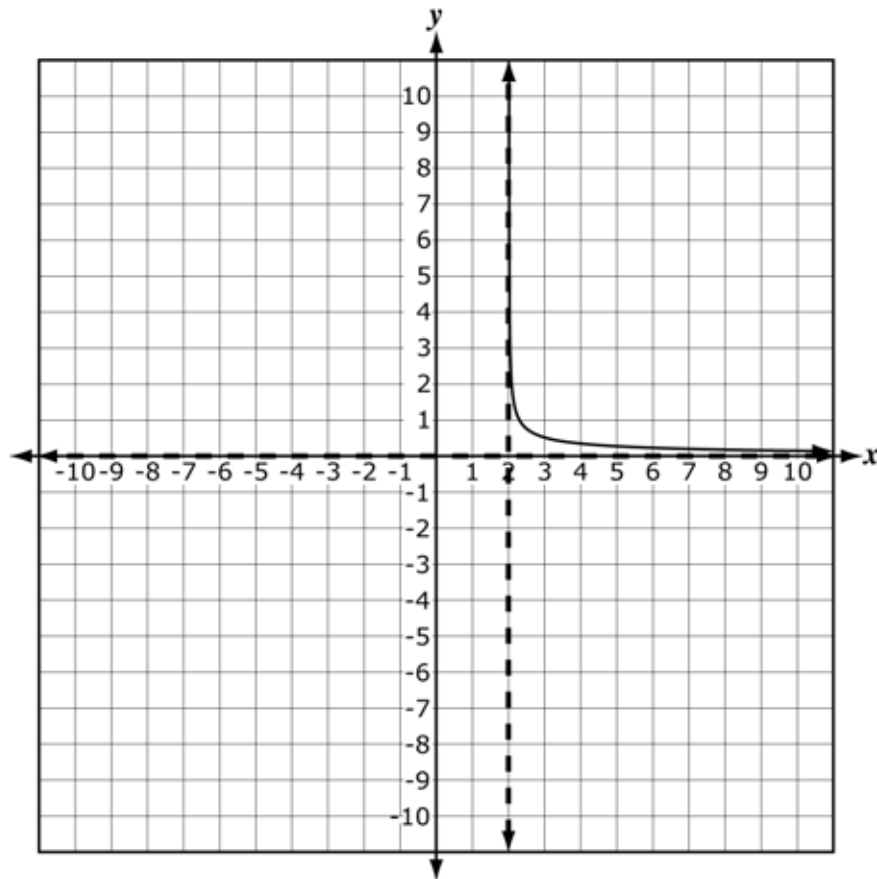
Students must first determine the composition of $k(m(x))$.

$$k(m(x)) = \frac{1}{\sqrt{3x-6}}$$

Then students must determine the restrictions on the domain of $k(m(x))$. This occurs for any x -value where the denominator of a fraction equals zero or is undefined. Thus, students must determine the x -values that make the denominator greater than zero to find the domain of $k(m(x))$ over the set of real numbers.

$$\begin{aligned}\sqrt{3x - 6} &> 0 \\ 3x - 6 &> 0 \\ 3x &> 6 \\ x &> 2\end{aligned}$$

Alternatively, students may graph $k(m(x))$ in a graphing calculator and observe that the graph has an asymptote at $x = 2$ and appears to the right of this asymptote. This would imply that the domain of the graph is $x > 2$.



Item 19

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O3.e Perform operations on rational expressions, including complex fractions.

Item:

00. Simplify $\frac{\frac{a-b}{b^2}}{\frac{a^2-b^2}{b}}$.
- A. $\frac{1}{ab-b^2}$
- B. $\frac{1}{ab+b^2}$
- C. $\frac{a^2-2ab+b^2}{b^2}$
- D. $\frac{a^3-a^2b-ab^2+b^4}{b^3}$

8604MC_2

Correct Answer: B

Explanation of Distractors:

A	Incorrectly factored $a^2 - b^2$ to $(a - b)(a - b)$.
C	Multiplied $\frac{a - b}{b}$ by $\frac{a - b}{b}$.
D	Did not take the reciprocal of $\frac{a^2 - b^2}{b}$, multiplying $\frac{a - b}{b^2}$ by $\frac{a^2 - b^2}{b}$.

Sample Solution:

To solve this item, students need to combine the skills of factoring and rewriting division as multiplication by the reciprocal.

$$\begin{aligned}\frac{\frac{a-b}{b^2}}{\frac{a^2-b^2}{b}} &= \frac{a-b}{b^2} \cdot \frac{b}{a^2-b^2} \\ &= \frac{a-b}{b^2} \cdot \frac{b}{(a+b)(a-b)} \\ &= \frac{1}{b(a+b)} \\ &= \frac{1}{ab+b^2}\end{aligned}$$

Item 20

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: P2.d Determine key characteristics of simple rational functions and their graphs.

Item:

00. The characteristics of $p(x) = ax^b$ are listed in the table below.

Characteristics of $p(x)$

Characteristic	$p(x)$
Domain	All real numbers except 0
Range	All real numbers except 0
As x approaches $+\infty$	$y < 0$ and approaches 0
As x approaches $-\infty$	$y > 0$ and approaches 0

What must be true about the values of a and b ?

- A. $a < 0$, $b < 0$, and b must be even
- B. $a < 0$, $b > 0$, and b must be even
- C. $a < 0$, $b < 0$, and b must be odd
- D. $a > 0$, $b < 0$, and b must be odd

2344MC_3

Correct Answer: C

Explanation of Distractors:

A	Considers the function as having an even degree.
B	Considers the function as having an even degree and a positive power.
D	Considers the function as having a positive leading coefficient.

Sample Solution:

This item allows students to draw on their understanding of the key characteristics of a rational function and its graph. For instance, since as x approaches $+\infty$, $y < 0$ and approaches 0, and as x approaches $-\infty$, $y > 0$ and approaches 0, they can conclude that the graph of $p(x)$ occurs in quadrants II and IV. The domain and range, with an understanding of the end behavior, provide evidence that the function is rational; therefore, b is negative. Since the domain and range of

$p(x)$ include all real numbers except 0, the graph includes asymptotes $x = 0$ and $y = 0$, which is characteristic of the graph of a rational function.

More specifically, students can conclude that b must be negative and odd in the rational form $p(x) = ax^b$ because rational functions having an odd degree occur in opposite quadrants. In addition, they can conclude that a must be negative in the rational form because the leading coefficient of a rational function must be negative in order for its graph to occur in the even quadrants.

$$a < 0, b < 0, \text{ and } b \text{ must be odd}$$

Item 21

Item Type: Short Answer

Calculator: Permitted

Benchmark: F3.c Recognize, express, and solve problems that can be modeled using absolute value, step, and other piecewise-defined functions. Interpret their solutions in terms of the context.

Item:

00. The amount of federal income tax a single person with a taxable income of \$77,100 or less must pay is listed below.

- 10% of taxable income up to \$7,825
- 15% of taxable income more than \$7,825 up to \$31,850
- 25% of taxable income more than \$31,850 up to \$77,100

Write a piecewise function to give the total amount of federal income tax a single person owes with a taxable income of x .

1684SA_S

Correct Answer:

$$f(x) = \begin{cases} 0.1x, & 0 < x \leq 7,825 \\ 0.15x - 391.25, & 7,825 < x \leq 31,850 \\ 0.25x - 3,576.25, & 31,850 < x \leq 77,100 \end{cases}$$

Scoring Rubric:

Score	Description
2	Student earns 2 points.
1	Student earns 1 point.
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

2 points

- 1 point for having all three pieces defined correctly [See below.]
- 1 point for having the domains correct for each piece [See below.]

$$\left[f(x) = \begin{cases} 0.1x, & 0 < x \leq 7,825 \\ 0.15x - 391.25, & 7,825 < x \leq 31,850 \\ 0.25x - 3,576.25, & 31,850 < x \leq 77,100 \end{cases} \right]$$

Sample Solutions:

The first sample solution, shown below, received a score of 2. It had all three pieces defined correctly. It also has the correct domain for each piece of the function. Since this is a calculator item that did not require the student to show their work, the response is correct without showing the steps of substituting the values of x into the function to determine the amount of tax already paid that needs to be added to the function at the next level. Notice the student circled the final answer even though the two functions are equivalent.

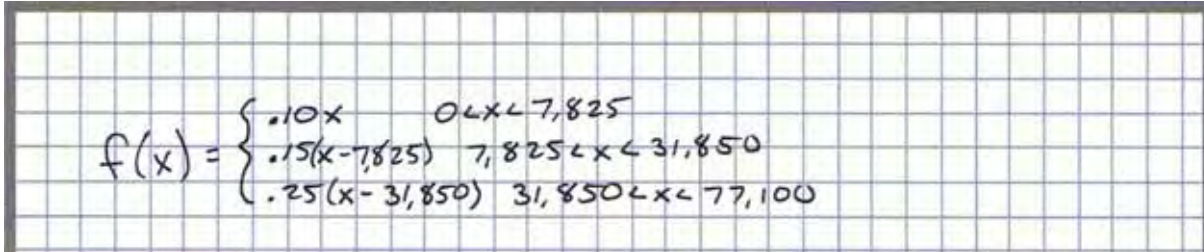
$$f(x) = \begin{cases} .10x & 0 \leq x \leq 7825 \\ .15(x - 7825) + 782.5 & 7825 < x \leq 31,850 \\ .25(x - 31,850) + 4386.25 & 31,850 < x \leq 77,100 \end{cases}$$
$$f(x) = \begin{cases} .10x & 0 \leq x \leq 7825 \\ .15x - 391.25 & 7825 < x \leq 31,850 \\ .25x - 3576.25 & 31,850 < x \leq 77,100 \end{cases}$$

This next sample received a score of 1. While it has the correct domain for each piece of the function defined correctly, only the expression for incomes up to \$7,825 is correct.

$$f(x) = \begin{cases} 0.1x & 0 \leq x \leq 7,825 \\ 0.15x & 7,825 < x \leq 31,850 \\ 0.25x & 31,850 < x < 77,100 \end{cases}$$

This final sample solution received a score of 0. The student had each part of the piecewise function partially correct, but did not include the constant, so the point for defining each piece correctly was not awarded. Also, while the numbers in the domains for each piece were correct,

the student used less than signs after x instead of less than or equal to signs, making the definitions of the domains incorrect.



A photograph of a student's handwritten work on a grid background. The student has written a piecewise function $f(x)$ with three cases. Each case uses a less-than sign (<) instead of a less-than-or-equal-to sign (<=).

$$f(x) = \begin{cases} .10x & 0 < x < 7,825 \\ .15(x - 7,825) & 7,825 < x < 31,850 \\ .25(x - 31,850) & 31,850 < x < 77,100 \end{cases}$$

Item 22

Item Type: Extended Response

Calculator: Permitted

Benchmark: Extended-response items can be written to address multiple aspects of the standard. This particular item was written to the P standard, *Polynomial and Rational Functions*, and mainly addresses the following benchmarks within the standard.

- P1.a Determine the key characteristics of quadratic functions and their graphs.
- P1.d Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.

Item:

00. A cell phone company predicts monthly profit using the equation

$P(x) = -0.6x^2 + 30x + 150$ where $P(x)$ is the monthly profit in thousands of dollars, and x is the amount spent on advertising in thousands of dollars.

Part A What amount should the company spend on advertising to maximize the monthly profit? Show or explain your work.

Part B Predict the maximum monthly profit. Show or explain your work.

Part C To the nearest dollar, what is the maximum amount the company can spend on advertising and still have a positive profit? Show or explain your work.

1218ER_5

Correct Answers:

Part A: \$25,000

Part B: \$525,000

Part C: \$54,580

Scoring Rubric:

Score	Description
4	Student earns 6 points.
3	Student earns 4 or 5 points.
2	Student earns 2 or 3 points.
1	Student earns 1 point.
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

Part A: 2 points

- 1 point for a correct strategy

$$\left[\begin{array}{l} x = \frac{-30}{2(-0.6)} \\ x = \frac{-30}{-1.2} \\ \text{or equivalent} \end{array} \right]$$

- 1 point for the correct amount [\$25,000]

Part B: 2 points

- 1 point for a correct strategy

$$\left[\begin{array}{l} P = -0.6(25)^2 + 30(25) + 150 \\ P = -0.6(625) + 750 + 150 \\ P = -375 + 750 + 150 \\ \text{or equivalent} \end{array} \right]$$

- 1 point for the correct amount [\$25,000] or correct answer based on incorrect answer in Part A

Part C: 2 points

- 1 point for a correct strategy

$$\left[\begin{array}{l} -0.6x^2 + 30x + 150 > 0 \\ x = \frac{-30 \pm \sqrt{900 - 4(-0.6)(150)}}{2(-0.6)} \\ x = \frac{-30 \pm \sqrt{1,260}}{-1.2} \\ \text{or equivalent} \end{array} \right]$$

- 1 point for the correct amount [\$54,580]

Sample Solution:

This first sample response earned a score of 4. For Part A, both points were awarded as the student showed a correct strategy to find the x -value for the maximum of the function and the correct answer was found.

Both points were also awarded for Part B. The student used a correct strategy of finding $P(25)$. The strategy in the scoring notes has more work being shown than the student gave in this response, but since a calculator was permitted for this item, it would be reasonable for the student to enter the expression shown, which has 25 substituted for x , into the calculator to jump to the final answer of 525. The student also interpreted the answer correctly here, remembering that the value of the expression is the number of thousands of dollars.

Finally, the response also earned both points for Part C. While the strategy in the scoring notes uses the quadratic formula to determine the maximum value of x that generates a profit, the item itself does not specify a specific method for the student to use, therefore the strategy used here would be considered an equivalent strategy. In this response, the student explained how he or she found the answer by using the Calculate zero feature of the graphing calculator. Had the

student just said, "I graphed the function," this would not have been specific enough of an explanation to earn the strategy point. Once again, the student interpreted the result in the context of the problem.

Part A $D(x) = -0.6x^2 + 30x + 150$

$$x = \frac{-30}{2(-0.6)} = \frac{-30}{-1.2} = 25$$

$\$25,000$

Part B $P(25) = -0.6(25)^2 + 30(25) + 150 = 525$

$\$525,000$

Part C I graphed the equation $y = -0.6x^2 + 30x + 150$ on my calculator. I then used the Zero function with a left bound at $(54.255, 11.476)$ and a right bound at $(54.893, -11.177)$ and a guess of $(54.255, 11.476)$. According to the calculator, the zero was $(54.580399, 0)$

To the nearest dollar, this would be

$\$54,580.$

Next is a sample of a response receiving a score of 3. The response earned 0 points for Part A. The strategy that the student used was incorrect as he or she failed to multiply the coefficient of the squared term by 2 in the denominator. Also, the answer given for Part A is incorrect.

The response did earn both points for Part B. Even though the answer is wrong, the strategy used was correct. The strategy applied the incorrect answer he or she got in Part A in a correct way, substituting 50 in for x in $P(x)$, then performing the calculations correctly using this incorrect answer.

The response also earned both points for Part C. A correct strategy, the quadratic formula, was used correctly, and a correct answer was given.

Since the response earned 4 of the 6 possible points, the score for the item is 3 as specified in the rubric.

A. $x = \frac{-30}{-.6} = 50$
 $\boxed{\$50,000}$

B. $-.6(50)^2 + 30(50) + 150$
 $= -.6(2500) + 1500 + 150$
 $= -1500 + 1500 + 150$
 $= 150$
 $\boxed{\$150,000}$

C. $P = -.6x^2 + 30x + 150$
 $x = \frac{-30 \pm \sqrt{30^2 - 4(-.6)(150)}}{2(-.6)}$
 $= \frac{-30 \pm \sqrt{900 + 360}}{-1.2}$
 $= \frac{-30 \pm \sqrt{1260}}{-1.2}$
 $= -4.58 \text{ or } 54.58$
 $\boxed{\$54,580}$

This sample response earned a score of 2. In Part A, the response earned both points. A correct strategy was used to determine the x -value of the maximum, earning 1 point. While the answer given, 25, was not changed into thousands, the item refers to x as being in thousands of dollars and therefore the answer can be assumed to be in the same form.

In Part B, the student used a correct strategy, substituting 25 for x , but the answer given is not correct. So for Part B, the response earned 1 of the 2 points.

In Part C, the student used a strategy that involved factoring, but the strategy was not a correct way to solve the problem because the equation was set to -150 and not zero. Because the strategy was incorrect, the answer was incorrect as well. So, Part C received no points.

Since the response earned 3 out of 6 points, the score for the response was a 2.

A. $x = \frac{-3.0}{2(-.6)} = 25$

B. $P(25) = -.6(25)^2 + 30(25) + 150 = 600$

C. $0 = -.6x^2 + 30x + 150$
 $-150 = -.6x(x - 50)$
 $250 = x(x - 50)$
 $x = 250$ or $x - 50 = 250$
 $x = 300$
 $\$250$ and $\$300$

The sample response on the next page earned a score of 1. In Part A the response did not earn any points since the strategy to find the x -value of the maximum was not correct.

In Part B, the response earned 1 point for having a correct strategy, substituting the solution from Part A into the function $P(x)$. Even though the response shows $P(150)$ being evaluated correctly, it did not receive the point for a correct answer based on an incorrect answer because the sign of the answer was changed when stating the final answer.

Part C of the response did not earn any points. The student divided the answer given in Part B by the coefficient of the x term. This was not a correct strategy. And while the answer to the division is correct based on the numbers that were used, the strategy and answer do not relate to what is being assessed.

Because this response earned just 1 of the 6 possible points, it received a score of 1.

Part A \$150

150 is the y-intercept

$$\text{Part B } -.6(150)^2 + 30(150) + 150 = -8850$$

\$8,850

$$\text{Part C } \frac{8850}{30} = \$295$$

The final sample response earned a score of 0. In Part A, the student set the value of the function as 0 and solved it, combining the x - and x -squared-terms together. For Part B, the student picked the constant term from the function to be the answer. And Part C, the student indicated that the cell phone company will always have profit, ignoring that the function indicates that profit will become negative to the right of the x -intercept.

A. $-.6x^2 + 30x + 150 = 0$

$$29.4x + 150 = 0$$

$$29.4x = -150$$

$$x = 5.10$$

B. The maximum profit is \$150

C. They will always have profit no matter what they spend. The more advertising, the more customers they will have.

Item 23

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O3.d Perform operations on polynomial expressions.

Item:

00. Which expression shows $\frac{-4x^3 + 14x^2 - 6x}{-6x^2 + 36x - 54}$ in simplest form?

A. $-4x^3 - \frac{58}{27}$

B. $\frac{2x^2 - x}{3x - 9}$

C. $\frac{2}{3}x + \frac{7}{18}x + \frac{1}{9}x$

D. $\frac{2x^3 - 7x^2 + 3x}{3x^2 - 18x + 27}$

6128MC_2

Correct Answer: B

Explanation of Distractors:

A	Incorrectly divided like terms, simplifying $-4x^3 + \frac{14x^2}{-6x^2} - \frac{6x}{36x} - \frac{1}{54}$ to $-4x^3 - \left(\frac{7}{3} - \frac{1}{6} - \frac{1}{54}\right)$.
C	Rewrote the expression as $\frac{-4x^3}{-6x^2} + \frac{14x^2}{36x} + \frac{-6x}{-54}$ and simplified
D	Removed a common factor from both the numerator and denominator, but did not factor the resulting polynomials. $\frac{-4x^3 + 14x^2 - 6x}{-6x^2 + 36x - 54} = \frac{-2(2x^3 - 7x^2 + 3x)}{-2(3x^2 - 18x + 27)} = \frac{2x^3 - 7x^2 + 3x}{3x^2 - 18x + 27}$

Sample Solution:

One possible method for completely simplifying the expression is for students to factor the numerator and denominator, eliminate common factors between the numerator and denominator, and then multiply the remaining factors in the numerator and in the denominator to match choice B.

$$\begin{aligned}\frac{-4x^3 + 14x^2 - 6x}{-6x^2 + 36x - 54} &= \frac{-2x(2x^2 - 7x + 3)}{-6(x^2 - 6x + 9)} \\ &= \frac{-2x(2x - 1)(x - 3)}{-6(x - 3)^2} \\ &= \frac{x(2x - 1)}{3(x - 3)} \\ &= \frac{2x^2 - x}{3x - 9}\end{aligned}$$

Item 24

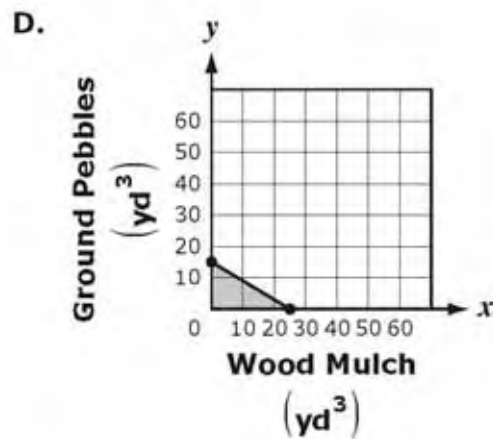
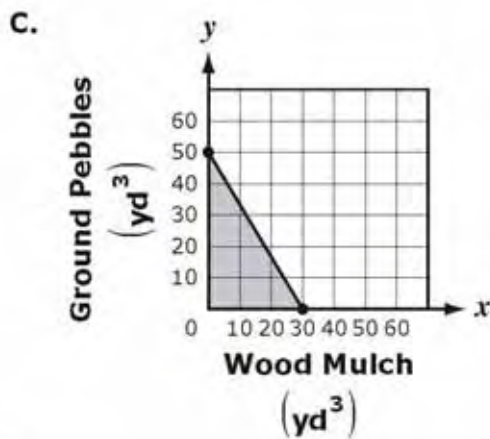
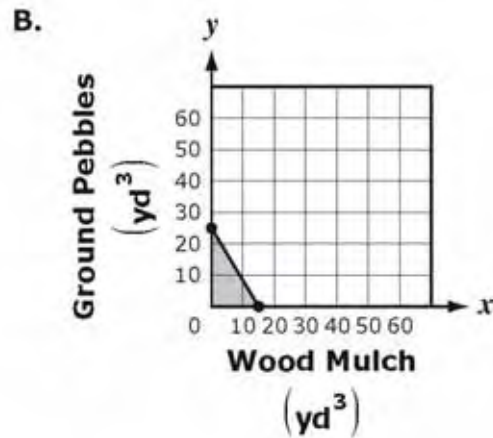
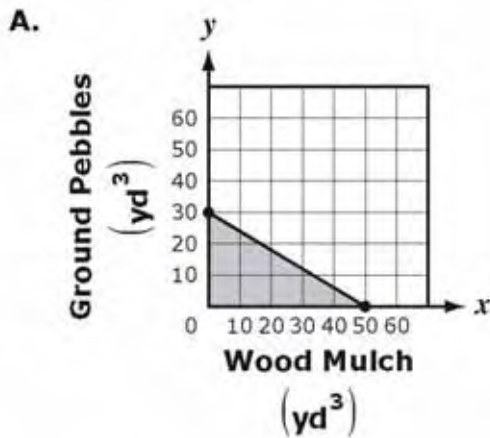
Item Type: Multiple Choice

Calculator: Permitted

Benchmark: E1.d Recognize and solve problems that can be represented by single variable linear equations or inequalities or systems of linear equations or inequalities involving two or more variables. Interpret the solution(s) in terms of the context of the problem.

Item:

00. Halley wants to buy two different types of ground coverings for her yard. She plans to spend no more than \$750 in all. She found wood mulch for \$30 per cubic yard and ground pebbles for \$50 per cubic yard, taxes included. Which of these graphs best represents the numbers of cubic yards of each type of ground covering Halley can buy?



8509MC_4

Correct Answer: D

Explanation of Distractors:

A	Used the price per cubic yard as the x - and y -intercepts of the boundary line, switching the two values.
B	Confused the x - and y -intercepts.
C	Used the price per cubic yard as the x - and y -intercepts of the boundary line.

Sample Solution:

In order to determine the correct graph for the situation presented in the problem, students can write a system of inequalities using x for the number of cubic yards of wood mulch and y for the number of cubic yards of ground pebbles that Halley can buy. She can buy no less than 0 cubic yards of either type of ground covering, so $x \geq 0$ and $y \geq 0$ are two of the inequalities in the system.

The third inequality uses the prices of the two types of ground covering. Since she has a maximum of \$750 to spend, the third inequality would show that the sum of the products of each type ground covering and its price is less than or equal to \$750.

$$x \geq 0$$

$$y \geq 0$$

$$30x + 50y \leq 750$$

The first two inequalities have the effect of restricting the domain to the first quadrant of the coordinate plane and the axes that bound it on the left and bottom. The region below and to the left of the boundary line of the third inequality is shaded.

Item 25

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: P1.a Determine key characteristics of quadratic functions and their graphs.

Item:

00. The roots of a quadratic function are $x = -2 \pm \sqrt{7}$. Find the x -value of the vertex of the corresponding parabola.

- A. -4
- B. -2
- C. -1
- D. 0

2232MC_2

Correct Answer: B

Explanation of Distractors:

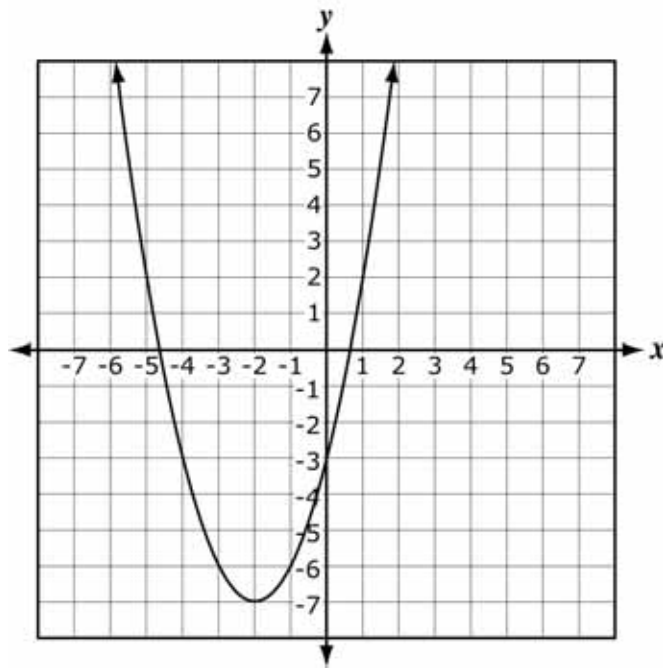
A	Adds the roots, but does not divide by 2. $-2 + \sqrt{7} + (-2 - \sqrt{7}) = -4$
C	Finds the midpoint between the x -intercepts, but divides it by 2.
D	Attempts to find the midpoint between the x -intercepts, but does not subtract correctly. $\frac{-2 + \sqrt{7} + (-2 - \sqrt{7})}{2} = \frac{0}{2} = 0$

Sample Solution:

Although there are multiple methods to correctly finding the x -value of the vertex of a parabola given its roots, understanding that the x -value of the vertex is located at the midpoint of the x -intercepts or the average of its roots is an important key characteristic of quadratics and their graphs. To correctly use this method to find the x -value of the vertex from the given roots, students must add them and then divide by 2.

$$\frac{(-2 + \sqrt{7}) + (-2 - \sqrt{7})}{2} = \frac{-4}{2} = -2$$

Another method students may choose is to find the quadratic function from the given roots, $y = (x + 2 - \sqrt{7})(x + 2 + \sqrt{7})$, and then graph it with a graphing calculator.



Students can then use the Calculate function of the graphing calculator to determine the minimum of the parabola, $(-2, -7)$.

Item 26

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: X1.b Represent exponential functions using tables, graphs, verbal statements, and equations. Represent exponential expressions in multiple forms. Translate among these representations.

Item:

00. The function listed below describes a family of exponential functions.

$$f(x) = 3^x + c$$

Which table corresponds to this family function?

A.

x	0	1	2	3	4
$f(x)$	1	4	7	10	13

B.

x	0	1	2	3	4
$f(x)$	2	4	10	28	82

C.

x	0	1	2	3	4
$f(x)$	1	2	9	28	65

D.

x	0	1	2	3	4
$f(x)$	2	3	5	9	17

3182MC_2

Correct Answer: B

Explanation of Distractors:

A	The values for $f(x)$ increase by a common difference of 3. The function that could represent this table of values is $f(x) = 3x + 1$.
C	The values for $f(x)$ increase by raising the corresponding x -value to the power of 3 and adding 1. The function that could represent this table of values is $f(x) = x^3 + 1$.
D	The values for $f(x)$ increase by raising 2 by the corresponding x -value and adding 1. The function that could represent this table of values is $f(x) = 2^x + 1$.

Sample Solution:

Students must recognize that if the successive differences in the values for $f(x)$ can be found by multiplying by a common ratio, then the relation between those values is exponential and that ratio is base of the function. Since the successive differences in the $f(x)$ values of the second table can be found by multiplying by 3, the function that could represent this table of values is $f(x) = 3^x + 1$, which is part of the given family of exponential functions.

Students could also determine the correct answer by entering $y = 3^x$ into a graphing calculator and viewing its table of values.

x	y
0	1
1	3
2	9
3	27
4	81

The y values in the table are closest to the corresponding $f(x)$ values in answer choice B except each is smaller by 1. This also indicates that the table of values belongs to the family of exponential functions $f(x) = 3^x + c$.

Item 27

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O2.b Perform operations on the set of complex numbers.

Item:

00. Given non-zero real numbers a and b , for which of the following values of x is $x(a+bi)$ a real number?

A. i^2

B. $-bi$

C. $a - bi$

D. $-(a+bi)$

6118MC_3

Correct Answer: C

Explanation of Distractors:

A	$i^2 = -1$
B	Adds instead of multiplying: $-bi + (a + bi) = a$
D	$-(a + bi)(a + bi) = -a^2 - 2abi + b^2$ (complex number)

Sample Solution:

Students should recognize that multiplying a complex expression by its conjugate results in a real number. They can verify this by substituting $a - bi$ for x and simplifying.

$$\begin{aligned}(a - bi)(a + bi) &= a^2 - abi + abi - b^2i^2 \\ &= a^2 - b^2(-1) \\ &= a^2 + b^2\end{aligned}$$

Since a and b are non-zero real numbers, $a^2 + b^2$ is real.

Item 28

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: X1.d Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.

Item:

00. The number of cells in a bacteria culture doubles every hour. If there were 500 bacteria cells in a culture at the start of an observation, how many hours will it take to have 50,000 cells in the culture?

- A. 1.70
- B. 2.35
- C. 6.64
- D. 9.97

9119MC_3

Correct Answer: C

Explanation of Distractors:

A	Divided 50,000 by the product of 500 and 2, taking the logarithm of the result.
B	Multiplied 500 and 2 together incorrectly, resulting in the equation $50,000 = 100^t$.
D	Divided 50,000 by 500 incorrectly, resulting in $1,000 = 2^t$.

Sample Solution:

There are multiple ways that students can solve this item. One way would be to solve it algebraically. To begin, students have write an equation to model the situation. Since the number of cells doubles each hour, an exponential equation can be used for the model. Since the initial number of bacteria cells was 500, the equation could be $B = 500(2)^t$, where B is the number of bacteria cells after t hours. Students then substitute 50,000 in the equation for B and solve for t .

$$50,000 = 500(2)^t$$

$$100 = 2^t$$

$$\log 100 = t \log 2$$

$$t = \frac{\log 100}{\log 2} \approx 6.64$$

Another method students can use is to solve the problem with a graph. To do this, students would have to determine an equation to model the situation, but in this case the student could use $y = 500(2)^x$ where y is the number of bacteria cells after x hours. Students can then graph this equation on a graphing calculator, setting the window large enough to show the function until y exceeds 50,000. One possible range the student could use would have x going from 0 to 10, counting by 1, and y from 0 to 60,000, counting by 10,000.

After graphing the equation, students can use the Trace function on the calculator to find the x -value when y is 50,000. Using the ranges described previously, the student would learn that x is about 6.60 when y is about 48,360 and x is about 6.70 when y is about 52,061. Therefore, x is between 6.60 and 6.70 when y is 50,000. Choice C is the only one of the four in this range.

Item 29

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: P1.c Describe the effect that changes in the parameters of a quadratic function have on the shape and position of its graph.

Item:

00. A quadratic function is represented by $f(x) = a(x - h)^2 + k$, where a , h , and k are non-zero real numbers. Which of the following changes will have no effect on the values of the real zeros for a function in this form?
- A. adding 2 to k
 - B. adding -1 to h
 - C. multiplying a by -1
 - D. multiplying the function by 2

3155MC_4

Correct Answer: D

Explanation of Distractors:

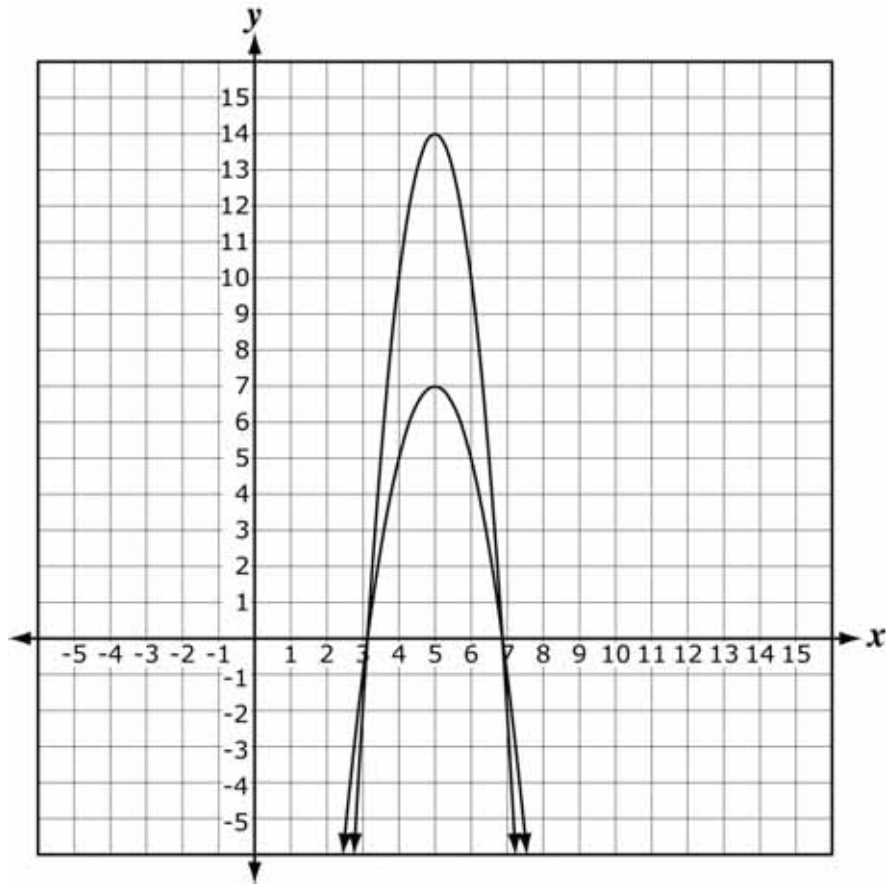
A	Adding 2 to k causes the graph to shift up 2 units, and the student incorrectly thinks this will never change the x -intercept(s).
B	Adding -1 to h causes the graph to shift left 1 unit, and the student incorrectly thinks this will never change the x -intercept(s).
C	Multiplying a by -1 in this form causes the graph to reflect across the horizontal line that passes through the vertex, and the student incorrectly thinks this will never change the x -intercept(s).

Sample Solution:

Students should understand that the zeros of a quadratic function are the x -intercepts of its graph. In addition, they should understand that in the quadratic form $f(x) = a(x - h)^2 + k$, (h, k) represents the vertex, the sign of a indicates the direction of the parabola, and $|a|$ indicates its magnitude. Thus, it is possible to add a value to the x - or y -coordinate of the vertex and not change the position of the x -intercepts in the case where there are none (for example, adding 1 to k if $f(x) = 2(x - 5)^2 + 7$).

However, counterexamples exist where adding a value to either coordinate of the vertex changes the position of the x -intercepts (for example, adding 1 to k if $f(x) = -2(x - 5)^2 + 7$). A similar argument can be made for the reflection of the parabola across the horizontal line that passes through the vertex and the position of its x -intercepts.

Multiplying an entire quadratic function by a non-zero real number other than 1 changes the magnitude and/or direction of the parabola but not the position of the x -intercepts. Students can explore this effect by graphing $f(x) = -2(x - 5)^2 + 7$ and $g(x) = 2(-2(x - 5)^2 + 7)$ in a graphing calculator.



Thus, multiplying the function by 2 does not have an effect on the values of the real zeros of a quadratic function in the given form.

Item 30

Item Type: Extended Response

Calculator: Permitted

Benchmark: Extended-response items can be written to address multiple aspects of the standard. This particular item was written to the X standard, *Exponential Functions*, and mainly addresses the following benchmarks within the standard.

X1.b Represent exponential functions using tables, graphs, verbal statements, and equations. Represent exponential expressions in multiple forms. Translate among these representations.

X1.d Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.

Item:

00. The average price of a gallon of gasoline in the United States on January 1, 2006, was \$2.238. On January 1, 2007, the average price of a gallon of gasoline was \$2.334.

Part A By what percent did the price increase from January 1, 2006, to January 1, 2007?

Part B Write a formula giving the price of a gallon of gasoline in terms of the number of years following 2006 if gasoline prices continue to increase exponentially at the annual rate found in Part A.

Part C Use your formula to predict the year when the price of a gallon of gasoline in the United States will reach \$10. Show or explain your work.

1665ER_5

Correct Answers:

Part A: 4.3%

Part B: $C(n) = 2.238(1.043)^n$

Part C: In the year 2041

Scoring Rubric:

Score	Description
4	Student earns 5 points.
3	Student earns 3 or 4 points.
2	Student earns 2 points.
1	Student earns 1 point.
0	Response is incorrect or irrelevant to the skill or concept being measured.
Blank	Student fails to respond.

Scoring Notes:

Part A: 1 point

- 1 point for a correct answer [4.3%]

Part B: 1 point

- 1 point for a correct answer $\left[C(n) = 2.238(1.043)^n \text{ or equivalent} \right]$ or a correct answer based on an incorrect answer in Part A

Part C: 3 points

- 1 point for a correct strategy

$$\left[\begin{array}{l} 10 = 2.238(1.043)^n \\ 4.468 = 1.043^n \\ \ln 4.468 = n \ln 1.043 \\ n = \frac{\ln 4.468}{\ln 1.043} \approx 35.6 \\ \text{or equivalent} \end{array} \right]$$

- or a correct strategy based on an incorrect exponential function in Part B
- 1 point for a correct solution to the equation $[n \approx 35.6]$ or a correct solution based on an incorrect exponential function in Part B
- 1 point for a correct answer [in the year 2041 or a correct answer based on an incorrect answer in Part A or an incorrect exponential function in Part B]

Sample Solutions:

The first sample response for this item received a score of 4. For Part A, the response shows a correct answer, so 1 point was earned.

In Part B, a correct exponential formula with the correct initial value of 2.238 and a correct base indicating growth of 4.3% was given to find the price, earning 1 point. A different name was used for the function than the scoring notes show, but no specific name was identified in the item, so this is considered correct.

Three points were awarded for Part C as there is a correct strategy shown, with a correct value of n , and a correct year. The answer that the student gave for n , 35.6, is obtained by not rounding the answer until the end of the process. It is then used to determine the year by adding it to 2006.

Because the response earned all 5 of the possible points, it received a score of 4 as indicated by the rubric.

Part A

$$\frac{2.334 - 2.238}{2.238} \approx .0429$$

about 4.3%

Part B

$$P(n) = 2.238(1.043)^n$$

n = # of years after 2006

Part C

$$10 = 2.238(1.043)^n$$

$$4.468 = (1.043)^n$$

$$\log 4.468 = \log 1.043^n$$

$$\log 4.468 = n \log 1.043$$

$$n = \frac{\log 4.468}{\log 1.043} \approx 35.6$$

$$2006 + 35.6 = 2041.6$$

About $\frac{6}{10}$ of the way through 2041.

The next sample response earned a score of 3. In Part A, the correct answer of 4.3% was given without any work shown. However, since work was not required, this earned 1 point for Part A.

In Part B, a correct equation was given, earning the 1 point for Part B.

For Part C, the student decided to graph the equation from Part B using the graphing calculator. The graph is not shown, but the explanation describes the student's strategy sufficiently enough to earn the point for a correct strategy. The response also earned the second point for Part C, finding a correct solution to the equation. However, the student did not follow through with answering the question about what year the price will reach \$10, so the third point for Part C was not earned.

Since the response earned 4 out of the 5 possible points, the response was given a score of 3.

A. 4.3%

B. $y = 2.238 \times 1.043^x$ $y = \text{price}$
 $x = \text{\# of years}$

C. 35.6 years

I graphed the equation on my graphing calculator with a window where x went from 0 to 50 and y from 0 to 11. I then traced the graph until $y = 10$. I found that when $y = 10$ $x = 35.6$.

This sample response earned a score of 2. For Part A, the response shows an answer of 4%. This answer was not considered precise enough for the given data. Although significant digits are not specifically assessed on the Algebra II exam, it is important for students to gain a general understanding of precision. In order to earn the point, the answer did not have to be written to 4 significant digits, but did need to be rounded to at least one decimal place. Therefore, the point was not earned for Part A.

The equation in Part B is incorrect since the student used .04 for the base of the exponent instead of 1.04. Using 1.04 would have been considered correct for this response since the answer the student gave for Part A was 4%. So, no point is earned for Part B either.

In Part C, the student wrote the equation, correctly substituting 10 for p even though the equation has an error based on the answer given in Part B. The steps used to solve the equation are correct and the answer to the exponential equation that was written was also correct. Therefore the response earned the 2 points for strategy and a correct solution to the equation. A year was not given though, so the third point for Part C was not earned.

Since the response only earned 2 of the 5 points, the student earned a score of 2.

Part A 4%

Part B $p = 2.238 (.04)^n$

Part C $10 = 2.238 (.04)^n$
 $4.468 = .04^n$
 $n = \frac{\log 4.468}{\log .04} = -.465$

This sample response earned a score of 1. For Part A, the response shows a correct percent increase of 4.3%, earning the 1 point for Part A.

In Part B, the response shows a linear equation written using numbers from the problem. Since this is not a correct equation, the response did not earn the point for Part B.

The equation from Part B was solved correctly in Part C, however to earn the points for a correct strategy or for a correct answer based on an incorrect equation in Part B, the equation in Part B must be exponential. Neither of these 2 points was earned. Furthermore, a year is not given for the answer, so that point was not earned either.

Since the response earned 1 of the 5 possible points, the response earned a score of 1.

Part A $\frac{2.334 - 2.238}{2.238} = 4.3\%$

Part B $P = 2.238 + .043t$

Part C $10 = 2.238 + .043t$
 $7.762 = .043t$
 $t = 180.5 \text{ years}$

The final sample response earned a score of 0. In Part A the student calculated the percent increase incorrectly, dividing by the wrong amount. This resulted in the first point not being awarded.

In Part B, the response shows an incorrect equation. The student added x to the power of the percent increase. Therefore, the point for Part B was not awarded either.

Since the equation in Part B was not exponential, the points in Part C could not be earned.

Since none of the possible points were earned, this response has a score of 0.

Part A $\frac{2.334 - 2.238}{2.334} = .0411 = 4.11\%$

Part B price = $2.334 + x \cdot 0411$

Part C $10 = 2.334 + x \cdot 0411$
 $7.666 = x \cdot 0411$
 $x = 186.5$