**TABLE OF CONTENTS**

**Unit Plan 7.NS.1-3 Operations with Rational Numbers ……………………. Pages 2 – 15**

**Lesson Plan 7.NS.1 Add and Subtract Rational Numbers ………………… Pages 16 – 34**

**Lesson Seed 7.NS.1b Adding Integers ……………………………………….. Pages 35 – 37**

**Lesson Seed 7.NS.1d Adding and Subtracting Rational Numbers ……… Pages 38 – 40**

**Overview:** *The overview statement is intended to provide a summary of major themes in this unit.*

This unit builds on prior understandings of addition, subtraction, multiplication and division of fractions.

This unit extends the understanding of addition, subtraction, multiplication and division of decimals and fractions to integers. They will find the absolute value of numbers, describe opposite quantities combined to make 0 as additive inverses, apply properties of operations as strategies to perform the four operations and converting fractions to decimals.

**Teacher Notes:** *The information in this component provides additional insights which will help educators in the planning process for this unit.*

* Students should be well-grounded in their knowledge of addition, subtraction, multiplication, and division of whole numbers.
* Students should have prior experience with positive and negative rational numbers.
* Students should have prior knowledge of properties of operations for whole numbers and be able to extend them to rational numbers.

**Enduring Understandings:** *Enduring understandings go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.*

At the completion of the unit on addition, subtraction, multiplication and division of rational numbers, the student will understand that:

* Rational numbers can be represented in multiple ways.
* Mathematical properties reveal multiple appropriate methods to compute.
* Rational numbers allow us to make sense of situations that involve numbers that are not whole.
* Rational numbers are ratios of integers.
* Two different integers can have the same absolute value.

***Essential* Question(s):** *A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.*

* Can you reverse the order of rational numbers when performing any operation and still get the same answer?
* How do operations with integers compare to operations with rational numbers?
* How does the opposite of *n* differ from the absolute value of *n*?
* How are properties useful in solving a variety of problems?

**Content Emphases by Clusters in Grade 7:** *According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), some clusters require greater emphasis than others. The list below shows PARCC’s relative emphasis for each cluster.**Prioritization does not imply neglect or exclusion of material. Clear priorities are intended to ensure that the relative importance of content is properly attended to. Note that the prioritization is in terms of cluster headings.*

 **Key: ■ Major Clusters**  **Supporting Clusters**  **Additional Clusters**

**Ratios and Proportional Reasoning**

**■**Analyze proportional relationships and use them to solve real-world and mathematical problems.

**The Number System**

**■ Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

**Expressions and Equations**

**■** Use properties of operations to generate equivalent expressions.

**■** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

**Geometry**

 Draw, construct, and describe geometrical figures and describe the relationships between them.

 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

**Statistics and Probability**

 Use random sampling to draw inferences about a population.

 Draw informal comparative inferences about two populations.

 Investigate chance processes and develop, use, and evaluate probability models.

**Focus Standards (Listed as Examples of Opportunities for In-Depth Focus in the PARCC Content Framework document):**

*According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), this component highlights some individual standards that play an important role in the content of this unit. Educators should give the indicated mathematics an especially in-depth treatment, as measured for example by the number of days; the quality of classroom activities for exploration and reasoning; the amount of student practice; and the rigor of expectations for depth of understanding or mastery of skills.*

**7.NS.3** When students work toward meeting this standard (which is closely connected to **7.NS.1** and **7.NS.2**), they consolidate their skill and understanding of addition, subtraction, multiplication, and division of rational numbers.

**Possible Student Outcomes:**

*The following list provides outcomes that describe the knowledge and skills that students should understand and be able to do when the unit is completed. The outcomes are often components of more broadly-worded standards and sometimes address knowledge and skills necessarily related to the standards. The lists of outcomes are not exhaustive, and the outcomes should not supplant the standards themselves. Rather, they are designed to help teachers delve deeply into the standards and augment as necessary, providing added focus and clarity for lesson planning purposes. This list is not intended to imply any particular scope or sequence.*

The student will be able to:

* represent addition and subtraction on a horizontal or vertical number line diagram.
* apply properties of operations to add, subtract, multiply and divide rational numbers.
* extend previous understandings to multiply and divide rational numbers.
* convert fractions into decimal form to determine if it terminates or repeats.
* Solve real-world problems using the four operations.

**Progressions from Common Core State Standards in Mathematics:** *For an in-depth discussion of the overarching, “big picture” perspective on student learning of content related to this unit, see:*

 The Common Core Standards Writing Team (10 September 2011). *Progressions for the Common Core State Standards in Mathematics* *(draft),* accessed at: <http://ime.math.arizona.edu/progressions/>

**Vertical Alignment*:*** *Vertical curriculum alignment provides two pieces of information:*

* *A description of prior learning that should support the learning of the concepts in this unit*
* *A description of how the concepts studied in this unit will support the learning of additional mathematics*
* **Key Advances from Previous Grades:** In grade 6, students learned about rational numbers and the kinds of quantities they can be used to represent; students also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will use the properties of operations, opposites, absolute value, additive inverses, number line diagrams, and changing fractions to decimals in order to add, subtract, multiply, and divide within the system of rational numbers.
* **Additional Mathematics:** Students will use addition, subtractions, multiplication, and division of rational numbers:
* in grade 8 when expanding their knowledge in rational to irrational numbers
* in algebra and geometry when they expand into real numbers and beyond

**Possible Organization of Unit Standards:** *This table identifies additional grade-level standards within a given cluster that support the overarching unit standards from within the same cluster. The table also provides instructional connections to grade-level standards from outside the cluster.*

| **Overarching Unit Standards** | **Supporting Standards** **within the Cluster** | **Instructional Connections** **outside the Cluster** |
| --- | --- | --- |
| **7.NS.1:** Apply and extend previous understandings of addition and subtraction to add and subtract **rational numbers**, and represent addition and subtraction on a horizontal or **vertical number line** diagram. | **7.NS.1a:*** Describe situations in which opposite quantities combine to make 0.

 **7.NS.1b:*** Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are **additive inverses**). Interpret sums of **rational numbers** by describing real-world contexts.

**7.NS.1c:*** Understand subtraction of rational numbers as adding the **additive inverse**, *p* – *q* = *p* + (–*q*). Show that the distance between two rational numbers on the number line is the **absolute value** of their difference, and apply this principle in real-world contexts.

**7.NS.1d:*** Apply properties of operations as strategies to add and subtract **rational numbers**.
 | **7.EE.1:** Apply properties of operations as strategies to add, subtract, **factor**, and **expand linear expressions** with rational coefficients.**7.EE.2:** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.  |
| **7.NS.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.  | **7.NS.2a:*** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

**7.NS.2b:*** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then:

 $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$Interpret quotients of **rational numbers** by describing real-world contexts.**7.NS.2c**:* Apply properties of operations as strategies to multiply and divide **rational numbers**.

**7.NS.2d:*** Convert a **rational number** to a decimal using long division; and know that the decimal form of a rational number terminates in 0s or eventually repeats.
 |  |
| **7NS.3:** Solve real-world and mathematical problems involving the four operations with **rational numbers**.  |  | **7.EE.3:** Solve multi-step real-life and mathematical problems posed with positive and negative **rational numbers** in any form (whole numbers, fractions, and decimals), using tools strategically; apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.**7.EE.4:** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.**7.EE.4a:*** Solve word problems leading to equations of the form *px* + *q* = *r* and *p*(*x* + *q*) = *r*, where *p*, *q*, and *r* are specific **rational numbers**; solve equations of these forms fluently; compare an **algebraic solution** to an **arithmetic solution**, identifying the sequence of the operations used in each approach.

**7.EE.4b:*** Solve word problems leading to inequalities of the form *px* + *q* > *r* or *px* + *q* < *r*, where *p*, *q*, and *r* are specific **rational numbers**; graph the solution set of the inequality and interpret it in the context of the problem.
 |

**Connections to the Standards for Mathematical Practice:** *This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice.  These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.*

In this unit, educators should consider implementing learning experiences which provide opportunities for students to:

1. **Make sense of problems and persevere in solving them.**
* Analyze a problem and depict a good way to solve the problem.
* Consider the best way to solve a problem.
* Interpret the meaning of their answer to a given problem.
1. **Reason abstractly and quantitatively**
* Consider the notion that addition, subtraction, multiplication and division of fractions can be represented in more than one way.
* Decide whether or not their answer connects to the question?
1. **Construct Viable Arguments and critique the reasoning of others.**
* Justify the process of working with addition, subtraction, multiplication and division of fractions to answer a question.
* Justify an argument using estimation with positive and negative benchmark fractions.
1. **Model with Mathematics**
* Draw a diagram that represents addition and subtraction of positive and negative fractions.
* Analyze an authentic problem and use a nonverbal representation of the problem.
* Use appropriate manipulatives to represent operations with positive and negative fractions.
1. **Use appropriate tools strategically**
* Use virtual media and visual models to explore word problems involving addition, subtraction, multiplication and division of fractions.
1. **Attend to precision**
* Demonstrate their understanding of the mathematical processes required to solve a problem by communicating all of the steps in solving the problem.
* Label positive and negative fractions appropriately.
* Use the correct mathematics vocabulary when discussing problems.
1. **Look for and make use of structure.**
* Look at a representation of addition, subtraction, multiplication and division of fractions and recognize the relationship that is represented in each.
* Compare, reflect, and discuss multiple solution methods.
1. **Look for and express regularity in reasoning**
* Pay special attention to details and continually evaluate the reasonableness of their answers.
* Use mathematical principles that will help them in solving a problem.

**Content Standards with Essential Skills and Knowledge Statements and Clarifications:** *The Content Standards and Essential Skills and Knowledge statements shown in this section come directly from the Maryland State Common Core Curriculum Frameworks. Clarifications were added as needed. Educators should be cautioned against perceiving this as a checklist. All information added is intended to help the reader gain a better understanding of the standards.*

| **Standard** | **Essential Skills and Knowledge** | **Clarification** |
| --- | --- | --- |
| **7.NS.1:**  Apply and extend previous understandings of addition and subtraction to add and subtract **rational numbers**, and represent addition and subtraction on a horizontal or **vertical number line** diagram.**1a:**  Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.**1b:**  Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are **additive inverses**). Interpret sums of **rational numbers** by describing real-world contexts.**1c:**  Understand subtraction of rational numbers as adding the **additive inverse**, *p* – *q* = *p* + (–*q*). Show that the distance between two rational numbers on the number line is the **absolute value** of their difference, and apply this principle in real-world contexts.**1d:** Apply properties of operations as strategies to add and subtract **rational numbers**.  | * **Ability to build on prior experience with positive and negative rational numbers (see 6.NS.5)**
* **Ability to identify additive inverses using rational numbers**
* **Knowledge of positive or negative values for fractions and decimals**
* **Ability to build on prior experience with absolute value (see 6.NS.7)**
* **Knowledge of absolute value to add and subtract rational numbers using a horizontal or a vertical number line**
* **See the skills and knowledge that are stated in the Standard.**
* **Ability to identify and apply the following properties:**
* **Commutative Property of Addition**
* **Associative Property of Addition**
* **Identity Property of Addition**
 | ***rational numbers:*** Numbers that can be expressed as an integer, as a quotient of integers (e.g., $\frac{1}{2}$, $\frac{4}{3}$, 7, - $\frac{1}{2}$, - $\frac{4}{3}$ , ‾ 7 ), or as a decimal where the decimal part is either finite or repeats infinitely (e.g., 2.75, ‾2.75, 3.3333… and ‾3.3333…) are considered rational numbers.A vertical number line is a scale with positive values located in ascending order above the origin (0) and negative values located in descending order below the origin (as on the y-axis of the coordinate plane), as opposed to a horizontal number line with positive values to the right of the origin and negative values to the left of the origin (as on the x-axis of the coordinate plane).***vertical number line:*** http://t0.gstatic.com/images?q=tbn:ANd9GcS1crkSGT7hWnL71GWSE2HjC8QjRFflhHauHrTPgg09cK0p8lWmAw yyx***additive inverse:*** The additive inverse of a number *a* is the number *‾a* for which *a* + (*‾a*) = 0.***absolute value:*** The absolute value of a number *a*, written $\left|a\right|$, is the non-negative number which is equal to *a* if *a* is non-negative, and equal to *‾a* if *a* is negative. Example: $\left|3\right|$ = 3; $\left|0\right|$ = 0; and $\left|‾ 3\right|$ = 3.  |
| **7.NS.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.  **2a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.**2b**: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then: $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$Interpret quotients of **rational numbers** by describing real-world contexts.**2c:** Apply properties of operations as strategies to multiply and divide **rational numbers**.**2d:** Convert a **rational number** to a decimal using long division; and know that the decimal form of a rational number terminates in 0s or eventually repeats. | * **Ability to identity and apply the following properties:**
	+ **Multiplicative Inverse**
	+ **Commutative Property of Multiplication**
	+ **Associative Property of Multiplication**
	+ **Identity Property of Multiplication**
	+ **Recognize that rules for multiplying signed numbers remain the same for all rational numbers**
* **Ability to explore and to justify the result of division by 0 (zero)**
* **Ability to apply and extend knowledge of addition and subtraction of integers (i.e., two color counters, arrows on a number line) to multiplication and division**
* **Ability to use patterns and concrete models to devise a general rule for dividing integers**: $-\left(\frac{p}{q}\right) = \frac{-p}{q}= \frac{p}{-q}$

$-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$* **Ability to Identify and apply the following properties:**
	+ **Distributive Property**
	+ **Associative Properties**
	+ **Commutative Properties**
	+ **Identity Properties**
* **Ability to recognize that when rational numbers in fractional form are converted to decimals, they either terminate or repeat**
 |  |
| **7NS.3:** Solve real-world and mathematical problems involving the four operations with **rational numbers**. (Note: Computations with **rational numbers** extend the rules for manipulating fractions to **complex fractions**.) | * **Ability to describe and identify complex fractions (see 7.RP.1)**
* **Ability to apply knowledge of Order of Operations**
 | ***complex fraction*:**  A complex fraction has a fraction for the numerator or denominator or both.***order of operations:*** The steps for simplifying expressions are:1. compute inside grouping symbols, including parentheses ( ), brackets [ ], braces { }, and above or below a division bar —―2. compute with exponents 3. multiply and divide in order from left to right4. add and subtract in order from left to right |

**Evidence of Student Learning:** *The Partnership for Assessment of Readiness for College and Careers (PARCC) has awarded the Dana Center a grant to develop the information for this component. This information will be provided at a later date. The Dana Center, located at the University of Texas in Austin, encourages high academic standards in mathematics by working in partnership with local, state, and national education entities.  Educators at the Center collaborate with their partners to help school systems nurture students' intellectual passions.  The Center advocates for every student leaving school prepared for success in postsecondary education and in the contemporary workplace.*

**Fluency Expectations/Recommendations:** *This section highlights individual standards that set expectations for fluency, or that otherwise represent culminating masteries. These standards highlight the need to provide sufficient supports and opportunities for practice to help students meet these expectations. Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.*

* + **7.NS.1–2** Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

**Common Misconceptions:** *This list includes general misunderstandings and issues that frequently hinder student mastery of concepts regarding multiplication and division of fractions.*

 Students may:

* misunderstand mathematical terms such as additive inverses, absolute value, properties of operations, terminating and complex fractions.
* incorrectly use the rules for signs of rational numbers.
* confuse the use of different properties of operations.
* not understand how to work with complex fractions.

**Interdisciplinary Connections:** *Interdisciplinary connections fall into a number of related categories:*

* *Literacy standards within the Maryland Common Core State Curriculum*
* *Science, Technology, Engineering, and Mathematics standards*
* *Instructional connections to mathematics that will be established by local school systems, and will reflect their specific grade-level coursework in other content areas, such as English language arts, reading, science, social studies, world languages, physical education, and fine arts, among others.*

**Model Lesson Plan Chart**

|  |
| --- |
| **Available Model Lesson Plans** |
| The lesson plan(s) have been written with specific standards in mind.  Each model lesson plan is only a MODEL – one way the lesson could be developed.  We have NOT included any references to the timing associated with delivering this model.  Each teacher will need to make decisions related to the timing of the lesson plan based on the learning needs of students in the class. The model lesson plans are designed to generate evidence of student understanding. This chart indicates one or more lesson plans which have been developed for this unit. Lesson plans are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings.  |
| **Standards Addressed** | **Title/Description** | **Suggested Use** |
| 7.NS.1-3 | Operations with Rational Numbers |  |

**Model Lesson Seed Chart**

|  |
| --- |
| **Available Lesson Seeds** |
| The lesson seed(s) have been written with specific standards in mind.  These suggested activity/activities are not intended to be prescriptive, exhaustive, or sequential; they simply demonstrate how specific content can be used to help students learn the skills described in the standards. Seeds are designed to give teachers ideas for developing their own activities in order to generate evidence of student understanding.This chart indicates one or more lesson seeds which have been developed for this unit. Lesson seeds are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings.  |
| **Standards Addressed** | **Title** | **Description/Suggested Use** |
| 7.NS.1b | Adding Integers |  |
| 7.NS.1d | Adding and Subtracting Rational Numbers |  |

**Sample Assessment Items:** *The items included in this component will be aligned to the standards in the unit and will include:*

* *Items purchased from vendors*
* *PARCC prototype items*
* *PARCC public release items*
* *Maryland Public release items*
* *Formative Assessment*

**Interventions/Enrichments/PD: (***Standard-specific modules that focus on student interventions/enrichments and on professional development for teachers will be included later, as available from the vendor(s) producing the modules.)*

**Vocabulary/Terminology/Concepts:** *This section of the Unit Plan is divided into two parts. Part I contains vocabulary and terminology from standards that comprise the cluster which is the focus of this unit plan. Part II contains vocabulary and terminology from standards outside of the focus cluster. These “outside standards” provide important instructional connections to the focus cluster.*

**Part I – Focus Cluster: Apply and Extend Previous Understandings of Operations with Fractions to Add, Subtract, Multiply, and Divide Rational Numbers**

***rational numbers:*** Numbers that can be expressed as an integer, as a quotient of integers (e.g., $\frac{1}{2}$, $\frac{4}{3}$, 7, - $\frac{1}{2}$, - $\frac{4}{3}$ , ‾ 7 ), or as a decimal where the decimal part is either finite or repeats infinitely (e.g., 2.75, ‾2.75, 3.3333… and ‾3.3333…) are considered rational numbers.

***additive inverse:*** The additive inverse of a number *a* is the number *‾a* for which *a* + (*‾a*) = 0.

***absolute value:*** The absolute value of a number *a*, written $\left|a\right|$, is the non-negative number which is equal to *a* if *a* is non-negative, and equal to *‾a* if *a* is negative. For example: $\left|3\right|$ = 3; $\left|0\right|$ = 0; and $\left|‾ 3\right|$ = 3.

***complex fraction*:**  A complex fraction has a fraction for the numerator or denominator or both.

***order of operations:*** The steps for simplifying expressions are:

1. compute inside grouping symbols, including parentheses ( ), brackets [ ], and braces { }, and above or below division bar ——

2. compute with exponents

3. multiply and divide in order from left to right

4. add and subtract in order from left to right

***vertical number line:***

A vertical number line is a scale with positive values located in ascending order above the origin (0) and negative values located in descending order below the origin (as on the y-axis of the coordinate plane), as opposed to a horizontal number line with positive values to the right of the origin and negative values to the left of the origin (as on the x-axis of the coordinate plane).



y

x

**Part II – Focus Clusters: Use Properties of Operations to generate Equivalent Expressions; and**

 **Solve Real-Life and Mathematical Problems Using Numerical and Algebraic Expressions and Equations**

***factor:*** A factor is a term that divides a given quantity evenly (with a remainder of 0). As a verb, factor means to resolve (divide) a given quantity in the form of its factors. For example, 6 is factored in the form of 2 x 3. The terms 2 and 3 are factors of the given quantity 6. 4*x*3 – 5*x* 2 is factored in the form of *x* 2 (4*x*– 5). The terms *x* 2 and (4*x*– 5) are the factors of 4*x*3 – 5*x* 2.

***expand linear expression:*** The form a quantity takes when written as a continued product, using the distributive property of multiplication over addition. For example, the quantity *x* 2 (4*x*– 5) in expanded form is 4*x*3 – 5*x* 2.

***linear expression:*** A linear expression can be comprised of a variable, a number, or a combination of variables, numbers, and operation symbols such that the exponents for all variables in the expression are limited to 0 and 1. For example: $ x^{0}$, y (or $y^{1}$), 16, 47 + 19, 2x – 3.

***properties of operations:*** The properties of operations apply to the rational number system, the real number system, and the complex number system, when *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties include:

associative property of addition (*a* + *b*) + *c = a* + (*b* + *c*)

commutative property of addition *a* + *b = b* + *a*

additive identity property of 0 *a* + *0 = 0* + *a = a*

existence of additive inverses *a* + *(–a) = (–a)* + *a = 0*

associative property of multiplication *(a* x *b)* x *c = a* x *(b* x *c*)

commutative property of multiplication *a* x *b = b* x *a*

multiplicative identity property of 1 *a* x *1 = 1* x *a = a*

existence of multiplicative inverses *a* x *1/a = 1/a* x *a = 1, a ≠ 0*

distributive property of multiplication over addition *a* x(*b* +*c*) *= a* x *b* + *a* x *c*

***algebraic solution:*** An algebraic solution is a proof or an answer that uses letters (algebraic symbols) to represent numbers, and uses operations symbols to indicate algebraic operations of addition, subtraction, multiplication division, extracting roots, and raising to powers.

***arithmetic solution:*** An arithmetic solution is a proof or an answer that uses rational numbers under the operations of addition, subtraction, multiplication and division .

**Resources:** *This section contains links to materials that are intended to support content instruction in this unit.*

* <http://ime.math.arizona.edu/progressions/>
* Additive Inverse Video: <http://teachertube.com/>
* <http://www.educationalrap.com/song/inversion.html>

| **Lesson Plan: 7.NS.1: Add and Subtract Rational Numbers***(This lesson should be adapted, including instructional time, to meet the needs of your students.)* |
| --- |
| **Background Information** |
| Content/Grade Level | The Number System/7th Grade  |
| Unit/Cluster  | Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. |
| Essential Questions/Enduring Understandings Addressed in the Lesson | Why do additive inverses always equal 0?How do operations with integers compare to operations with rational numbers? Understand authentic contexts when opposite quantities of rational numbers are combined. Rational numbers allow us to make sense of situations that involve numbers that are not whole. |
| Standards Addressed in This Lesson | **7.NS.1a.** Describe situations in which opposite quantities combine to make 0.**7.NS.1b.** Understand *p* and *q* as the number located a distance $\left|q\right|$ from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.**7.NS.1d.** Apply properties of operations as strategies to add and subtract rational numbers. It is critical that the **Standards for Mathematical Practice** are incorporated in ALL lesson activities throughout the unit as appropriate.  It is not the expectation that all eight Mathematical Practices will be evident in every lesson.  The **Standards for Mathematical Practice** make an excellent framework on which to plan your instruction.  Look for the infusion of the Mathematical Practices throughout this unit. |
| Lesson Topic | Describe authentic, student-centered situations in which opposite quantities combine to make 0; add and subtract rational numbers. |
| Relevance/Connections | **7.NS.3.** Solve real-world and mathematical problems involving the four operations with rational numbers. |
| Student Outcomes | * Students will be able to identify and use additive inverses when adding and subtracting rational numbers.
* Students will be able to apply their understanding of additive inverse to authentic scenarios.
 |
| **Background Information** |
| Prior Knowledge Needed to Support This Learning | Prior knowledge of:* addition and subtraction
* vertical and horizontal number lines
 |
| Method for determining student readiness for the lesson | See attached readiness assessment, *OMNIPRESENT RATIONALS!!!* This activity is a quick check to determine student familiarity with addition and subtraction, and with reading number lines. Methods for student assessment might include: pinch strips (see attached directions), white boards, sign language, interactive polling program (e.g., Turning Point), e-instruction device systems, among others).  |

| **Learning Experience** |
| --- |
| ***Component*** | ***Details*** | ***Which Standards of Mathematical Practice does this address? How is each Practice used to help students develop proficiency?*** |
| ***Warm Up*** | How are addition and subtraction different? How are they alike? Provide students with two color counters. Challenge students to: Explain a way to represent a situation where the color counters cancel each other out. |   |
| ***Motivation*** | Describe the game “Four Corners.” Students will be given four authentic situations that they may have explored in other content areas. The teacher should create one poster to describe each situation. The posters should be displayed in the four corners of the classroom. Students are given time to read all situations, choose the one of greatest personal interest, and then move into that corner. The situations for this activity are intended to be open-ended. Ask students to discuss the role of additive inverses from their personal knowledge of the given topics, such as: * Finances - Balancing a budget or to paying off credit card bills.
* Sports – Playing or scoring in a sport.
* Geography – Lines of longitude and latitude and their distances from the Prime Meridian and Equator, respectively.
* Geography – Measuring distances above versus below sea level.
* Teacher can decide to create another scenario that incorporates the experiences/cultures of students in the classroom.
 |   |
| ***Activity 1*** UDL Components* Multiple Means of Representation
* Multiple Means for Action and Expression
* Multiple Means for Engagement

Key QuestionsFormative AssessmentSummary | UDL Components:* Principle I: Representationis present in the activity. The Warm-up task incorporates options for perception by assigning colors for emphasis, e.g., positive values versus negative values. Options for comprehension are provided in the Motivation and in the activity through the use of authentic, student-centered scenarios that model key concepts regarding integers and additive inverses.
* Principle II: Expression is present in the activity. Students physically interact with instructional materials by examining note cards, taking turns placing the cards in categories on the chalk/white board, and then physically indicating their agreement or disagreement with a hand signal. This task also encourages students to “stop and think” before choosing the correct category and prompts them to categorize.
* Principle III: Engagement is present in the activity. Students initially work in small collaborative groups, creating small communities of learners engaged in common interests/activities. The degree of difficulty within the activity is differentiated through the use of cards with varying levels of complexity.

Directions:Develop understandings of additive inverse using multiple representations. See Activity #1: Concept Attainment. * Explain that students will categorize expressions, symbols, and word problems based on an unknown rule (expressions of additive inverse simplify to 0).
* Model categorizing note cards into two categories, “YES” (demonstrates additive inverse) or “NO” (does not demonstrate additive inverse). Scaffold lesson by initially sharing the more obvious cards, if necessary.
* Give students the same set of note cards (either individually or in pairs) to finish categorizing together.
* After students have finished their sorting, have each pair of students go to the board and place one of the note cards under the categories. After each card is added, discuss with the class whether they agree or disagree via “Thumbs Up – Thumbs Down.”
* Discuss the reasoning of agreement or disagreement. Continue for the remainder of the cards. By this point student should be trying to determine the rule.
* An alternative method is to have students work completely on their own to develop various rules that they see and then guide discussion toward the meaning of additive inverse.

*Guiding Questions:* * Explain your reasoning for placing the note card in a particular column.
* Can I change this card so that it will fit in the other category? If so, how would I change it? If not, why not?
 | * Make sense of problems and persevere in solving them by requiring students to identify and compare relationships in a variety of decontextualized and contextualized representations. (SMP #1)
* Construct viable arguments and critique the reasoning of others by engaging students in a discussion of why they agree or disagree with other students’ answers, and by expecting students to respond to the guiding questions in the activity. (SMP #3)
* Look for and make use of structure by expecting students to apply a rule for using additive inverses to solve problems, and by analyzing the overall structure of the examples in the concept attainment activity. (SMP #7)
 |
| ***Activity 2***UDL Components* Multiple Means of Representation
* Multiple Means for Action and Expression
* Multiple Means for Engagement

Key QuestionsFormative AssessmentSummary | UDL Components:* Principle I: Representationis present in the activity.

Combined with the Warm-up and Motivation, this activity presents students with an explicit opportunity for spaced review and practice with integers and additive inverses. * Principle II: Expression is present in the activity. This task encourages students to “stop and think” before choosing the correct category and prompts them to categorize. Moreover, the activity provides alternatives in the requirements for rate and timing allowed to interact with the instructional material on the posters.
* Principle III: Engagement is present in the activity. Students can choose to work in small collaborative groups, or independently as they complete the Gallery Walk and the think-pair-share summary task; thus, prompting students on when and how to ask peers and the teacher for assistance. In addition, the sources of information are contextualized to learners’ lives/experiences as Maryland residents.

 Directions:*Traveling Around Maryland Gallery Walk* (see ten attached questions answer sheet to post around classroom) * Make copies of the ten questions about Maryland towns/cities and post the ten problems around the classroom.
* Direct students to walk around to each of the posted questions in the gallery walk. Use attached answer sheet to compute and record the answer to each question.
* Some of the answers will equal 0, others will not.
* Calculators may be used, if necessary.
* Check answers from Gallery Walk as a class. Have each group share one response (under the document camera or an overhead, calling on a student, etc.)

Discussion after the completion of Gallery Walk: * What generalizations can you make about your answers?
* What was your answer for Salisbury? What were the steps that you used to arrive at your answer? If all students had the answer $4.85, is there a way that you could only have $3.60 left?
* How are your answers for the cities similar? How are they different? Explain.
* What would the sum be of Ocean City and St. Mary’s? Let’s analyze this: 90 + (-90). What does the symbol in the middle tell you about what you’re going to do with these numbers? What is the relationship between negative 90 and positive 90? Example answer: they are the opposite. What’s another word for opposite? Example answer: inverse. Therefore, with these numbers we are working with the additive inverse, just like in the concept attainment activity. (A number line may be used to model these numbers for a visual representation).

Summary: Think, Pair, Share Activity* Explain how the concept of additive inverse applies to having a balanced budget.
* Explain how a football player who rushes 15 yards, is pushed back 22 yards, and rushes7 yards, ends up in the same spot.
* Explain why par is a good score in golf?
* Explain how the measures for two lines of longitude or two lines of latitude can be combined to equal zero.
* Explain how a measure of altitude can be combined with a measure of depth to equal sea level.

Formative Assessment: Use the student conversations during the gallery walk and the responses to the questions asked to determine the depth of student understanding of additive inverse.  | * Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary. (SMP #1)
* Reason abstractly and quantitatively by requiring students to make sense of the quantities and relationships in each gallery walk problem. (SMP #2)
* Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve authentic problems, and then to reflect on whether their results are sensible for the given scenario. (SMP #4)
* Use appropriate tools strategically by expecting students to use estimation and other mathematical knowledge to identify potential errors. (SMP #5)
* Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning. (SMP #6)
* Look for an express regularity in repeated reasoning by expecting students to understand the broader application of additive inverses, and then recognize the same structure in similar situations from the gallery walk. (SMP #8)
 |
| ***Activity 3***UDL Components* Multiple Means of Representation
* Multiple Means for Action and Expression
* Multiple Means for Engagement

Key QuestionsFormative AssessmentSummary | UDL Components:* Principle I: Representationis present in the activity.

It provides students with a visual flow-chart diagram in place of the more usual equations. * Principle II: Expression is present in the activity. The chart is created in such a way that provides more than one avenue for determining the correct answer. Students can use graphing calculators to support their computation efforts.
* Principle III: Engagement is present in the activity. Students are given immediate feedback, which is aimed at supporting them in their progress in a timely and understandable manner. Options for sustaining student effort and persistence are introduced through differentiated degrees of difficulty in values included with the online activity.

 Directions:See the attached *Activity #3: Rational Flow Chart* document. * Direct students to look at the given values on the flow chart.
* They should begin the chart by combining given values to determine the values missing from lined spaces and within arrows.
* Similarly, the values students determine will provide clues needed to find additional, missing values.
* Students should build on their work until the flow chart is complete and the flow of values is correct.

Review the activity by asking guiding questions:* What strategy did you use to determine the missing values?
* Did your first strategy work? Why or why not?
* Is there only one solution for each combination of values? Explain your reasoning.
* How do you know your missing values are correct?
 | * Make sense of problems and persevere in solving them by expecting students to plan a strategy for completing the flow chart, and by monitoring their progress and reassessing their answers, if necessary. (SMP #1)
* Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another. (SMP #2)
* Use appropriate tools strategically by expecting students to estimate to check their responses and discover possible errors. (SMP #5)
* Attend to precision by requiring students to understand the meaning of operations symbols and signed quantities, as well as to calculate efficiently and correctly. (SMP #6)
* Look for and express regularity in repeated reasoning by understanding the overall process of the activity, yet still attending to the details of each computation. (SMP #8)
 |
| ***Activity 4***UDL Components* Multiple Means of Representation
* Multiple Means for Action and Expression
* Multiple Means for Engagement

Key QuestionsFormative AssessmentSummary | UDL Components:* Principle I: Representationis present in the activity.

Use of online manipulatives activates students’ prior knowledge through visual imagery, concept anchoring, and concept mastery.* Principle II: Expression is present in the activity. It encourages students to “stop and think” before choosing the correct answer. The activity provides alternatives in the requirements for rate and length of time students are allowed to interact with the online instructional material. Also, the activity provides option for physical action by using keyboard commands for mouse actions.
* Principle III: Engagement is present in the activity. Students are encouraged to persist through the spontaneity and novelty inherent in the computer game.

 Directions:Go to the National Library of Virtual Manipulatives website: <http://nlvm.usu.edu>* Click on **Number Operations 6-8** and then click on the *Circle Zero* game.
* Use this game to reinforce the rule for combining opposite quantities to make zero. This can be done individually, in partners, or as a whole group.
* What strategy did you use to place the numbers? Did your first strategy work? Why or why not?
* Is there only one solution for each circle? Explain your reasoning.
 | * Make sense of problems and persevere in solving them by expecting students to see relationships between quantities and analyze given information in order to plan a path of “attack” and solve the problem. (SMP #1)
* Reason abstractly and quantitatively by requiring students to make sense of the given quantities in the circles and their relationships with each other and with possible solution values. (SMP #2)
* Use appropriate tools strategically by expecting students to use estimation to and other mathematical knowledge to determine potential errors. (SMP #5)
* Attend to precision by requiring students to calculate accurately before they can continue with the next example in the game. (SMP #6)
* Look for an express regularity in repeated reasoning by providing students with an opportunity to understand the broader application of patterns and observe that structure in similar situations. (SMP #8)
 |
| ***Closure*** | Exit ticket: Describe an authentic situation that we have not discussed during the lesson in which opposite quantities combine to make zero. NOTE: Teachers may prefer to create an exit ticket that focuses on addition and subtraction of rational numbers.Students will complete the exit ticket independently. |  |

|  |
| --- |
| **Supporting Information** |
| Interventions/Enrichments* Students with Disabilities/Struggling Learners
* ELL
* Gifted and Talented
 | Intervention Activity: Additive Inverse Video from <http://teachertube.com/>Activity also can be used as a warm-up, review, or ending activity.Gifted and Talented Activity: Listen to rap song at <http://www.educationalrap.com/song/inversion.html>. Instruct students to create their own rap lyrics; combine original lyrics with a song.ELL: Ask students to draw picture examples of the concept of additive inverse and/or create Frayer model of examples/definitions |
| Materials | *OMIPRESENT RATIONALS!!!* DocumentDirections for Pinch StripsChart paper to post the Gallery Walk problems | Gallery Walk problems, Student Answer Sheet, and KeyConcept Attainment Cards and Key*RATIONAL FLOW CHART* document and Key |
| Technology | CalculatorsLCDDocument Camera | OverheadElectronic polling device (for example: Turning Point)Timer |
| Resources | Additive Inverse Video - <http://teachertube.com/><http://www.educationalrap.com/song/inversion.html> |

**OMNIPRESENT RATIONALS!!!**

1) Which point on the line shows 3 ½?



A

B

C

D

2) Which point on the line shows -2.5?



A

B

C

D

3) Which point on the line would show 3 degrees below zero?



5) An elevator is on Level 14. A person enters it and goes down 9 levels. Where is the elevator after it goes down 9 levels?

|  |  |
| --- | --- |
| a. 23b. 6 | c. 5d. 9 |

4) A pelican is flying 6 feet above the ocean. It dives 10 feet to catch a fish. How many feet below sea level is the fish?

|  |  |
| --- | --- |
| a. 0b. 4 | c. 10d. 16 |

B

C

D

A

**Directions for Pinch Strips (Cards)**

1. Print out a class set of these sheets. If laminated, they could be used for multiple years.
2. Tri fold the pinch strips (cards) so that the ABCD and EFGH sides are folded out. The operation symbols are on the inside. Teacher can choose which side of the pinch strip to use. ABCD for basic multiple choice. EFGH for H.S.A. multiple choice questions. Operation symbols for vocabulary/word problem questions where teachers can ask students: “What operation would you use first to solve this problem?”
3. When projecting the question on the overhead, SMART board, chalk board or even reading the question out loud, have the students pinch the letter choice answer when it is a multiple choice assessment.
4. Students hold up their response for the teacher to view.
5. Teacher can see a quick glimpse of who gets it and who does not.

Visual Representation of Folding Paper

1. 2) 3)

E

A

+

-

F

B

G

C

x

÷

H

D

 Back Side of Paper Front View Back View

|  |  |  |
| --- | --- | --- |
| A | E | + |
| B | F | − |
| C | G | x |
| D | H | ÷ |

**Activity 1: Note Card Template for Concept Attainment: -- cut apart**

|  |  |
| --- | --- |
| -2+2  | -5+6 |
|  | 3.427 − 3.247  |
|  214.1 − 214.1 | The highest altitude measured in the United States is Mt. McKinley, Alaska at 20,320 feet. The lowest altitude in the United States is Death Valley, California and is 282 feet below sea level.  |
| ‾$π+π$ | *a − b* (where *a* and *b* do not equal zero)  |
| At 8 A.M the temperature was 27. By 12 P.M. it rose 15F. By 8 P.M. the temperature was back down to 27.  |  |
| -1+**-1****+1** **-1** -1 | $$π+π$$ |
| Michele ran 3 miles on the treadmill and burned 200 calories. When she finished she ate a 200 calorie fun size bag of candy. | The Redskins gained 5 yards in the first down, and lost 3 yards in the second down.  |
| The start-up costs of a lemonade stand business is $13.50. A cup of lemonade costs 50. You sell 27 cups of lemonade. | A hot air balloon rises 2,150.825 feet. It then falls 583.37 feet. |
| Hole #7 at Minnie’s Miniature Golf is a par 3 hole. You took 3 putts to get the ball in the hole.  | Hole #12 at Minnie’s Miniature Golf is a par 3 hole. You got a hole in one. |
| Donte’s family budgets $375.50 forback to school clothes and supplies.After 2 days of shopping, Donte’sfamily spent $310.25 on clothes and $65.25 on supplies. | Jill’s credit card bill for the month of December was $983.76. She made a payment of $768.39. |
| Jen had $150.35 on her debit card. She went to the mall and spent $98.99 on three dresses and $52.98 on shoes. | An airplane took off from an airport and climbed to a height of 15,000 feet. After flying for one hour, the plane returned to the airport and landed. |
| Henry likes to scuba dive in the ocean. He dove 35 feet to look at fish and then dove 24 more feet to explore a coral reef. After an hour, Henry returned to the surface of the ocean. | A wild turtle entered a pond and swam on the surface of the water for 20 minutes. Afterwards, it climbed out and sunned itself on a log.(maybe… why or why not?) |

**Activity #1: Concept Attainment Answer Key**

**CONCEPT: Expressions that exemplify additive inverse**

|  |  |
| --- | --- |
| **YES** | **NO** |
| -2+2  | -5+6 |
|  | 3.427 – 3.247  |
|  214.1-214.1 | The highest altitude measured in the United States is Mt. McKinley, Alaska at 20, 320 feet. The lowest altitude in the United States is Death Valley, California and is -282 feet below sea level.  |
| -$π+π$ | a-b (where a and b do not equal zero)  |
| At 8 A.M the temperature was 27 . By 12 P.M. it rose 15 F. By 8 P.M. the temperature was back down to 27 .  |  |
|  +**-1****+1** | $$π+π$$ |
| Michele ran 3 miles on the treadmill and burned 200 calories. When she finished she ate a 200 calorie fun size bag of candy.  | The Redskins gained 5 yards in the first down, and lost 3 yards in the second down.  |
| It costs $13.50 to open a lemonade stand. A cup of lemonade costs $.50. You sell 27 cups of lemonade. | A hot air balloon rises 2,150.825 feet. It then falls 583.37 feet. |
| Hole number 7 at Minnie’s Miniature Golf is a par 3 hole. You took 3 putts to get the ball in the hole. | Hole number 12 at Minnie’s Miniature Golf is a par 3. You got a hole in one. |
| Donte’s family budgets $375.50 for back to school clothes and supplies. After 2 days of shopping they spent $310.25 on clothes and $65.25 on supplies. | Jill’s credit card bill for the month of December was $983.76. She made a payment of $768.39. |
| An airplane took off and climbed to a height of 15,000 feet. After flying for one hour, the plane returned to the airport and landed. | Jen had $150.35 on her debit card. She went to the mall and spent $98.99 on three dresses and 52.98 on shoes. |
| Henry likes to scuba dive in the ocean. He dove 35 feet to look at fish and then dove 24 more feet to explore a coral reef. After an hour, Henry returned to the surface of the ocean. | A wild turtle entered a pond and swam on the surface of the water for 20 minutes. Afterwards, it climbed out and sunned itself on a log.(maybe… why or why not?) |

**Activity # 2**

**Gallery Walk Problems**

***[The following are included as blackline masters with one problem on a single page. They were reduced for printing for this session in the interest of conserving paper.]***

**COLUMBIA**

In Columbia, the local government owns a total of 5,300 acres. Of this land, 950 acres are public parks, 3,780 acres are lakes, and 530 acres are ponds. The Symphony Woods comprise 40 acres. How many acres of government-owned land are unaccounted for?

**EASTON**

Last year, the November Waterfowl Festival hosted 15,000 visitors. This year, the festival hosted 2,078 senior citizens, 8,766 adults, and 4,156 children ages 12 and under. What is the difference in attendance between the two years?

**FREDERICK**

Of the 52 covered bridges in Maryland, Frederick County originally was home to eight bridges. Of those eight covered bridges, two were destroyed by fire; one was destroyed by storms; and five have been deemed unsafe due to neglect. How many bridges in Frederick County are still useable?

**BALTIMORE**

The Baltimore Ravens started a play on their 45 yard line. They rushed for 15 yards. The New York Jets pushed them back 22 yards on the next play. Then the Ravens completed a pass for 7 yards. Where on the field are the Ravens now?

**SALISBURY**

A student at Salisbury University has $12 in her food plan account. She bought pizza for $3.75, two cookies for $1.25, and a sports drink for $2.15. Now, the student wants to lend her friend $4.85 for lunch. Will the student have enough money on her account to lend her friend $4.85? Why or why not?

**OCEAN CITY**

In July, the early morning temperature of sand on the beach is usually about 50ºF. By 3 p.m., the sand temperature can go up to 140 ºF. What is the difference in temperatures?

**ST. MARY’S**

A submarine on its way to Patuxent River Naval Air Station is cruising at a depth of 161 feet in the Chesapeake Bay. The submarine rises 80.5 feet every ten minutes. What is the new depth of the submarine in twenty minutes?

**Annapolis**

Bob caught a 630-pound White Marlin while Tom caught fifteen, 42-pound rockfish. Which scale below represents the weight comparison of Bob’s fish versus Tom’s total rockfish catch, if Bob’s marlin is in the left pan and Tom’s catch is in the right pan?

1.  b.  c. 

Explain your reasoning.

**Potomac**

The distance from the Maryland coastline to Las Vegas is 2,560 miles. The distance from the coastline to Potomac is 107 miles. What is the distance from Potomac, MD to Las Vegas, NV?

**Cumberland**

Cumberland, Maryland is at an elevation of 627 feet. The Wisp Ski Resort, 45 miles away, has a maximum elevation of 3,080 feet. What is the difference in elevation?

Student Answer Sheet: ***Traveling Around Maryland Gallery Walk***

|  |  |
| --- | --- |
| Columbia | Easton |
| Frederick | Baltimore |
| Salisbury | Ocean City |
| St. Mary’s | Potomac |
| Cumberland | Annapolis |

Teacher Worksheet:

|  |  |
| --- | --- |
| Columbia = 0  | Easton = 0  |
| Frederick = 0  | Baltimore = 45 |
| Salisbury = Yes, she will have $4.85 left over.  | Ocean City = 90  |
| St. Mary’s = Sea Level (0) | Potomac = 2,453  |
| Cumberland = 2,453  | Annapolis = Scale is equal, B  |

RATIONAL FLOW CHART

RATIONAL FLOW CHART

 ‾6 \_\_ \_\_

+‾ 102

−‾221

−3.25

−‾108

−121.6

\_\_ 0 \_\_

+0.5

+‾8.7

 ­‾9.75 1.7 ‾17.3



\_\_

KEY: RATIONAL FLOW CHART

KEY

KEY

**Lesson Seed: Adding Integers**

*(Lesson seeds are ideas for the domain/cluster/standard that can be used to build a lesson.*

*An effective lesson plan requires more components than presented in a lesson seed.)*

|  |
| --- |
| **Domain: The Number System** **Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.****Standard: 7.NS.1b - Understand *p+q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.**  |
| **Purpose/Big Idea:*** To allow students to discover the mathematical rule for adding integers with different signs.
 |
| **Materials:*** Colored chips, or related manipulative (used for first few examples only)
* Graphic organizer (attached) – can be used inside of a sheet protector with dry erase markers (optional)
* Optional – Sheet protectors, dry erase markers, and erasers/paper towels in which to place ***The Zero Circle*** sheets for students to write on
 |
| **Activity: The Zero Circle** * This activity is a bridge from using manipulatives to add integers with different signs to creating a mathematical rule for adding integers with different signs.
* Complete ***Attachment #1: The Zero Circle*** to help students discover the rules for adding integers with different signs.
* For practice, teachers can provide students with additional examples to use with ***The Zero Circle***
* If necessary, teachers can provide time for students to work with manipulatives to show addition of integers.
* Note: Teacher should include real-world contexts once students have discovered the mathematical rule for adding integers with different signs
 |
| **Guiding Questions:** * After completing several examples, ask students: What do you notice? What are some generalizations you can make about the activity?
* Other possible questions to explore with students (reference – (book) *Good Questions for Math Teaching*; Grades 5-8 by Schuster and Anderson, Math Solutions, Sausalito, California, USA, 2005):
	+ For homework, Kim was adding integers. She looked at a computation and said, “I know the sign of the sum will be negative.” Based on this statement, what do you know about the computation? (This question allows students to generalize about the relationship between the sign of the sum and the numbers in an integer addition problem.)
	+ Jonathan added three integers and got a sum of zero. What might the integers have been?
 |

**Attachment # 1: The Zero Circle**

**Example A: -43 + 54**

Steps 1 and 2: Guiding Questions: Looking at the two integers to be added, decide which has the smaller absolute value (-43). In other words, thinking back to when you used colored chips, are there fewer red or black chips (there are fewer red/negative chips)? Since all of the chips of that color need to be removed, place this number into the zero circle.

-43

-43 + 54

Step 3: Guiding Questions: The value inside the zero circle must equal zero, therefore, the number with the opposite sign must be pulled from the other number in the original expression (as in removing chips). What is the opposite of -43 such that, when added, the value of the zero circle is zero? (+43)

-43 + 43

-43 + 54

Note to Teacher: This second arrow represents removing/pulling positive 43 (43 black/positive chips) from the original positive 54 (54 black/positive chips).

Step 4: Guiding Questions: The number that remains after this process/removal is the sum. After completing several examples, ask students: What do you notice? Can you make any generalizations? (The sum of two integers with different signs is the **difference** between the absolute values of the larger and smaller numbers, and gives the sum the sign of the original number with the larger absolute value.)

11 [Final answer: positive 11 (11 black positive chips).]

-43 + 54

= 0

= 0

= 11

-43 + 43

**Example B: -125 + 99**

Steps 1 and 2: Guiding Questions: Looking at the two integers to be added, decide which has the smaller absolute value (99). In other words, thinking back to when we used colored chips, are there fewer red or black chips (there are fewer black/positive chips)? Since all of the chips of that color need to be removed, place this number into the zero circle.

-125 + 99

 99

Step 3: Guiding Questions: The value inside the zero circle must equal zero, therefore, the number with the opposite sign must be pulled from the other number in the original expression (as in removing chips). What is the opposite of 99 such that, when added, the value of the zero circle is zero? (-99)

-125 + 99 54

 -99 + 99

 Note to Teacher: This second arrow represents removing/pulling -99 (99 red/negative chips) from the original -125 (125 red/negative chips).

Step 4: Guiding Questions: The number that remains after this process/removal is the sum. After completing several examples, ask students: What do you notice? Can you make any generalizations? (The sum of two integers with different signs is the **difference** between the absolute values of the larger and smaller numbers, and gives the sum the sign of the original number with the larger absolute value.)

-26 [Final answer: -26 (26 red/negative chips).]

-125 + 99 54

 -99 + 99

= 0

= 0

= -26

**ZERO CIRCLE**

**ZERO CIRCLE**

**Lesson Seed: Adding and Subtracting Rational Numbers**

*(Lesson seeds are ideas for the domain/cluster/standard that can be used to build a lesson.*

*An effective lesson plan requires more components than presented in a lesson seed.)*

|  |
| --- |
| **Domain: The Number System** **Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.****Standard: 7.NS.1d - Apply properties of operations as strategies to add and subtract rational numbers.**  |
| **Purpose/Big Idea:**To communicate and defend one’s thinking when applying the following properties as strategies to add and subtract rational numbers:* Commutative property of addition
* Associative property of addition
* Identity property of addition

This activity can be used as a closure activity after instruction on identification and application of the number properties of addition. (approximately 15 minutes)This activity may also be used as a formative assessment.  |
| **Materials:*** Print ? copies of each color balloon (# of copies depends on # of students in each group). Six colored balloons/problems are attached (if color printer is not available, can print each balloon on colored paper).
* Discussion starters for student conversation (see attachment).
 |
| **Activity:** Block Party* Divide class into groups based on number of problems being used (teacher’s decision to create groups randomly or strategically in advance).
* Distribute one balloon to each student.
* Ask students to evaluate the given problem individually on the front of the balloon. On the back of the balloon students will write the number property(ies) used to evaluate the problem.
* Group students with the same color balloon to verify their answers and explanations. The “Block Party” begins! Students should stand and find one other student in the class with a different color balloon (“mingle” with others). Each student will share his/her problem with the other student and explain which number property(ies) was used as a strategy to add or subtract the rational numbers. Encourage students to ask questions of each other (see attached sheet, *Student Discussion Starters*, for ideas) and to justify their thinking.
* Students will switch partners to find another different color balloon about which they have not yet conversed. Each student will share his/her problem with the other and explain which number property(ies) was used as a strategy to add or subtract the rational numbers. Encourage open discussion between the students (see *Student Discussion Starters* for ideas).
* Students continue switching partners until they have talked about all of the problems.
* Teacher will conclude activity with discussion questions (see guiding questions below).

Answers:Blue: ‾3 (Commutative property of addition; other possible strategy is use of “nice” numbers)Red: $\frac{2}{3}$$\frac{2}{3}$ (Commutative property of addition; other possible strategy is use of zero pair)Orange: 5 $\frac{31}{40}$ (Associative property of addition) Yellow: 29.7 (Commutative property of addition, then Associative property of addition)Green: 0.2 (Commutative property of addition; other possible strategy is use of zero pair)Purple: 0 (Commutative property of addition; other possible strategy is use of zero pairs)Other possible variations of the “Block Party”:* Teacher can strategically group students based on balloon color. (Example: Teacher may group struggling learners in the blue group such that the teacher can give additional attention when needed.)
* Teacher can alter the problems on the balloons based on student ability level.
* “Block Party” can be applied to number properties of multiplication by changing the problems on the balloons.
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| **Guiding Questions:*** What properties did you recognize?
* How are properties useful in solving a variety of problems?
* How are the commutative and associative properties similar? Different?
* What common strategies were used to solve various problems? (possible answers: commutative property, associative property, identity property, creating zero pairs, making “nice” numbers, etc.)
* What problem of your own could you write in order to use one or more of the properties discussed today?
* What is something that you found interesting in your class work/homework today? Describe it.
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**STUDENT DISCUSSION STARTERS**

* What property(ies) did you use to evaluate the expression?
* What is your final answer?
* I agree with because .
* I disagree with because .
* Why did you choose to ?
* I do not understand . Could you please tell me again?

*[The following graphics were reduced from a full page for each balloon in order to conserve paper for printing.]*

**(-25.2 + 89.7) + (-34.8)**

0.3 – 1.9 – 0.3 + 2.1

$$\frac{1}{4}+ \left(-\frac{2}{3}\right)+ \frac{3}{4}- \frac{1}{3}$$

$$\left(3\frac{1}{4}+ \frac{3}{5}\right)+ \left(\frac{3}{10}+ 1\frac{5}{8}\right)$$

$$\frac{1}{4}+ \frac{2}{3}+ \left(-\frac{1}{4}\right)$$

-6 + 7 – 4