



## **Side-by-Side Comparison of the Common Core College and Career Readiness Standards for Mathematics (Draft September 21, 2009) with The American Diploma Project (ADP) Benchmarks for Mathematics (2004)**

### Purpose

The purpose of this side-by-side comparison is to provide information to the states regarding the alignment of the ADP Mathematics Benchmarks with the revised Common Core College and Career Readiness Standards for Mathematics. We hope this side-by-side proves helpful as the states formulate feedback on the quality and content of these standards. The standards will be revised based on the 30-day public review period.

### Background

The Common Core College and Career Readiness Standards for Mathematics are meant to describe the essential skills and knowledge students will need to be prepared for non-remedial college mathematics courses and will be prepared for training programs for career-level jobs. In addition, the Common Core standards are intended to be focused, clear, and internationally benchmarked. [Note: These Common Core standards will be followed by K-12 standards that provide greater detail about the expectations for students at each level.]

### Side-by-side comparison chart

The side-by-side comparison chart shows the correspondence between the Common Core College and Career Readiness Standards for Mathematics and the ADP Benchmarks for mathematics (2004). Following the side-by-side chart is a list of the ADP Benchmarks for which Achieve found no corresponding Common Core standard. [Note: Comparisons are a matter of professional judgment, and other experts may have different impressions. In addition, the strength of the match varies from standard to standard.]

### Organization of the respective documents

The Common Core College and Career Readiness Standards for Mathematics are organized into three interconnected parts: (1) A Standard for Mathematical Practice that describes characteristics, or habits of mind, of proficient mathematics students, (2) Ten Standards for Mathematical Content, and (3) Example Tasks. Each Standard for Mathematical Content contains Core Concepts, Core Skills and a narrative that provides a coherent understanding of the content. The standards are as follows:



1. Mathematical Practice
2. Number
3. Quantity
4. Expressions
5. Equations
6. Functions
7. Modeling
8. Shape
9. Coordinates
10. Probability
11. Statistics.

For purposes of this comparative analysis, both the Mathematical Practice Standard and the 10 Standards for Mathematical Content have been included in the side-by-side chart that follows.

In addition, the College and Career Readiness Standards for Mathematics document also includes Example Tasks. Over time, the collection of tasks will grow. The Example Tasks illustrate the range and variety of student performance that is expected. Example Tasks exist for Quantity, Expressions, Equations, and Modeling. For purposes of this side-by-side analysis with the ADP Benchmarks, the Explanatory Problems have not been used.

The **American Diploma Project Benchmarks**, published in 2004, are organized into the following strands:

- I. Number Sense and Numerical Operations
- J. Algebra
- K. Geometry
- L. Data Interpretation, Statistics and Probability

Benchmarks denoted with an asterisk (\*) are recommended for all students but required for those students who plan to take calculus in college. In addition, the ADP Benchmarks identify nine Mathematical Reasoning skills that are defined both separately and woven throughout the benchmarks in the four content domains. The ADP Benchmarks are



accompanied by sample tasks from postsecondary faculty and employers that illustrate what students will encounter in college and on the job. They may be found at [www.achieve.org](http://www.achieve.org).

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We hope this side-by-side comparison will be helpful to you as you consider the College and Career Readiness Standards for Mathematics. Please let us know if you have any questions.

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**American Diploma Project (ADP) Benchmarks Addressed in the Common Core College and Career Readiness Standards for Mathematics (Draft 9/21/09)**

Note: Underlined text in some ADP Benchmarks indicates the particular element of the benchmark that is addressed by corresponding Common Core standard

College and Career Readiness Standards for Mathematics	American Diploma Project (ADP) Benchmarks
<b>Core Concepts</b>	
<p><b>Mathematical Practice</b>            Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.</p> <p>Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement. These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures. Taken together with the Standards for Mathematical Content, they support productive entry into college courses or career pathways.</p>	
<b>Core Practices</b>	
<p><b>MP1. Attend to precision.</b>            Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies noting the assumptions made. They clarify definitions. They state the meaning of</p>	<p><b>MR4.</b> Using the special symbols of mathematics correctly and precisely.</p> <p><b>MR5.</b> Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations.</p>

College and Career Readiness Standards for Mathematics	American Diploma Project (ADP) Benchmarks
<p>the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. Rather than saying, “let <math>v</math> be speed and let <math>t</math> be time,” they would say “let <math>v</math> be the speed in meters per second and let <math>t</math> be the elapsed time in seconds from a given starting time.” They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits indicate unwarranted precision.</p>	<p><b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert a measurement using one unit of measurement to another unit of measurement.</p>
<p><b>MP2. Construct viable arguments.</b> Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.</p>	<p><b>K1.1.</b> Identify, explain the necessity of and give examples of definitions, axioms and theorems.</p>
	<p><b>MR1.</b> Using inductive and deductive reasoning to arrive at valid conclusions.</p>
	<p><b>MR3.</b> Understanding the role of definitions, proofs and counterexamples in mathematical reasoning; constructing simple proofs.</p>
<p><b>MP3. Make sense of complex problems and persevere in solving them.</b> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”</p>	<p><b>MR8.</b> When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results.</p>
	<p><b>MR2.</b> Using multiple representations (literal, symbolic, graphic) to represent problems and solutions.</p>
	<p><b>MR6.</b> Distinguishing relevant from irrelevant information, identifying missing information and either finding what is needed or making appropriate estimates.</p>
	<p><b>MR7.</b> Recognizing and <u>using the process of mathematical modeling</u>: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and <u>interpreting the solution in the context of the</u></p>

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	<u>original problem.</u>
<p><b>MP4. Look for and make use of structure.</b> Mathematically proficient students look closely to discern a pattern. For example, in <math>x^2 + 5x + 6</math> they can see the 5 as <math>2 + 3</math> and the 6 as <math>2 \times 3</math>. They recognize the significance of an existing line in a geometric figure and can add an auxiliary line to make the solution of a problem clear. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects. For example, by seeing <math>5 - 3(x - y)^2</math> as 5 minus a positive number times a square, they see that it cannot be more than 5 for any real numbers <math>x</math> and <math>y</math>.</p>	<p><b>MR9.</b> Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight.</p>
<p><b>MP5. Look for and express regularity in repeated reasoning.</b> Mathematically proficient students pay attention to repeated calculations as they carry them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through <math>(1, 2)</math> with slope 3, they might abstract the equation <math>(y - 2)/(x - 1) = 3</math>. Noticing the regularity in the way terms cancel in the expansions of <math>(x - 1)(x + 1)</math>, <math>(x - 1)(x^2 + x + 1)</math>, and <math>(x - 1)(x^3 + x^2 + x + 1)</math> leads to the general formula for the sum of a geometric series. As they work through the solution to a problem, proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</p>	<p><b>MR9.</b> Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight.</p> <p><b>J1.7.</b> * <u>Derive</u> and use the formulas for the general term and summation of finite arithmetic and geometric series; find the sum of an infinite geometric series whose common ratio, <math>r</math>, is in the interval <math>(-1, 1)</math>.</p>
<p><b>MP6. Make strategic decisions about the use of technological tools.</b> Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, ruler, protractor, graphing calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding and estimation strategically, attending to levels of precision, to ensure appropriate levels of approximation and to detect possible errors. They are able to use</p>	<p><b>14.1.</b> Use calculators appropriately and make estimations without a calculator regularly to detect potential errors.</p> <p><b>14.2.</b> Use graphing calculators and computer spreadsheets.</p>

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<p>these tools to explore and deepen their understanding of concepts.</p>	
<p><b>Number</b>            Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra:</p> <ul style="list-style-type: none"> <li>• Numbers can be added in any order with any grouping and multiplied in any order with any grouping.</li> <li>• Adding 0 and multiplying by 1 both leave a number unchanged.</li> <li>• All numbers have additive inverses, and all numbers except zero have multiplicative inverses.</li> <li>• Multiplication distributes over addition.</li> </ul> <p>Subtraction and division are defined in terms of addition and multiplication, so are also governed by these rules.</p> <p>The place value system bundles units into 10s, then 10s into 100s, and so on, providing an efficient way to name large numbers. Subdividing in a similar way extends this to the decimal system, which provides an address system for locating all real numbers on the number line with arbitrarily high accuracy. Place value is the basis for efficient algorithms, reducing much computation to single-digit arithmetic. Mental computation strategies also make opportunistic use of the rules of arithmetic, as when the product <math>5 \times 177 \times 2</math> is computed at a glance to obtain 1770, rather than methodically working from left to right.</p> <p>An estimate may be more appropriate than an exact value, for example, when you want to know the number of calories in a meal. Often a result is reported using fewer digits than were calculated. A mature number sense includes having rules of thumb about how much accuracy is appropriate and understanding that accuracy to more than a few decimal places often takes substantial effort. Estimation and approximation are</p>	

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<p>also useful in checking calculations.</p> <p>Rational numbers represented as fractions can be located on the number line by seeing them as numbers expressed in different units; for example, <math>\frac{3}{5}</math> is 3 units, where each unit is <math>\frac{1}{5}</math>. However, rational numbers do not fill out the number line. There are also irrational numbers, such as <math>\pi</math> or <math>\sqrt{2}</math>. Each point on the number line then corresponds to a real number that is either rational or irrational.</p> <p><i>Connections to Expressions, Functions and Coordinates.</i> The rules of arithmetic govern the manipulations of expressions and functions. Two perpendicular number lines define the coordinate plane.</p>	
<b>Core Concepts</b>	
<p><b>NA. The real numbers include the rational numbers and are in one-to-one correspondence with the points on the number line.</b></p>	<p><b>I2.1.</b> <u>Locate the position of a number on the number line</u>, know that its distance from the origin is its absolute value and know that the distance between two numbers on the number line is the absolute value of their difference.</p>
<p><b>NB. Quantities can be compared using division, yielding rates and ratios.</b></p>	<p><b>I1.2.</b> Calculate and apply ratios, proportions, rates and percentages to solve problems.</p>
	<p><b>I1.4.</b> Explain and apply basic number theory concepts such as prime number, factor, divisibility, least common multiple and greatest common divisor.</p>
<p><b>NC. A fraction can represent the result of dividing the numerator by the denominator; equivalent fractions have the same value.</b></p>	<p><i>The ADP Benchmarks are predicated on understandings students acquire prior to entering high school. The foundations of place value, rules of arithmetic, and fractional equivalence are presumed under ADP's I1.1 which calls for students to be able to compute with integers, fractions, and decimals.</i></p>
<p><b>ND. Place value and the rules of arithmetic form the foundation for efficient algorithms.</b></p>	

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<b>Core Skills</b>	
<p><b>N1. Compare numbers and make sense of their magnitude.</b>            Include positive and negative numbers expressed as fractions, decimals, powers, and roots. Limit to square and cube roots. Include very large and very small numbers and the use of scientific notation.</p>	<p><b>I2.</b> Recognize and apply magnitude (absolute value) and ordering of real numbers:</p> <p><b>I2.2.</b> Determine the relative position on the number line of numbers and the relative magnitude of numbers expressed in fractional form, in decimal form, as roots or in scientific notation.</p>
<p><b>N2. Know when and how to use standard algorithms, and perform them flexibly, accurately and efficiently.</b></p>	<p><b>I1.1.</b> Add, subtract, multiply and divide integers, fractions and decimals.</p> <p><b>I1.3.</b> Use the correct order of operations to evaluate arithmetic expressions, including those containing parentheses.</p> <p><b>I1.5.</b> Multiply and divide numbers expressed in scientific notation.</p>
<p><b>N3. Use mental strategies and technology to formulate, represent and solve problems.</b></p>	<p><b>I4.1.</b> Use calculators appropriately and make estimations without a calculator regularly to detect potential errors.</p>
<p><b>N4. Solve multi-step problems involving fractions and percentages.</b>            Include situations such as simple interest, tax, markups/markdowns, gratuities and commissions, fees, percent increase or decrease, percent error, expressing rent as a percentage of take-home pay, and so on.</p>	<p><b>I1.1.</b> Add, subtract, multiply and divide integers, fractions and decimals.</p> <p><b>I1.2.</b> Calculate and apply <u>ratios</u>, proportions, <u>rates and percentages</u> to solve problems.</p>
<p><b>N5. Use estimation and approximation to solve problems.</b>            Include evaluating answers for their reasonableness, detecting errors, and giving answers to an appropriate level of precision.</p>	<p><b>I4.1.</b> Use calculators appropriately and <u>make estimations without a calculator regularly to detect potential errors.</u></p> <p><b>MR5.</b> Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations.</p>
<p><b>Quantity</b>            A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.</p>	



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<p>The length of a football field and the speed of light are both quantities. If we choose units of miles per second, then the speed of light has a value of approximately 186,000 miles per second. But the speed of light need not be expressed in miles per second; it may be approximated by <math>3 \times 10^8</math> meters per second or in any other unit of speed. Bare numerical values such as 186,000 do not describe quantities unless they are paired with units.</p> <p>Speed (distance divided by time), rectangular area (length multiplied by length), density (mass divided by volume), and population density (number of people divided by land area) are examples of derived quantities, obtained by multiplying or dividing quantities.</p> <p>It can make sense to add two quantities, such as when a child 51 inches tall grows 3 inches to become 54 inches tall. To be added or subtracted, quantities must be of the same type (length, area, speed, etc.); to add or subtract their values, the quantities must be expressed in the same units. Converting quantities to have the same units is like converting fractions to have a common denominator before adding or subtracting. But, even when quantities have the same units it does not always make sense to add them. For example, if a wooded park with 300 trees per acre is next to a field with 30 trees per acre, they do not have 330 trees per acre.</p> <p>Doing algebra with units in a calculation reveals the units of the answer, and can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed.</p> <p><i>Connections to Number, Expressions, Equations, Functions, Modeling and Statistics.</i> Operations described under Number and Expressions govern the operations one performs on quantities, including the units involved. Quantity is an integral part of any application of mathematics, and has connections to solving problems using data, equations, functions and modeling.</p>	

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<b>Core Concepts</b>	
<b>QA. The value of a quantity is not specified unless the units are named or understood from the context.</b>	<b>K8.1.</b> Understand that <u>numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions</u> ; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert a measurement using one unit of measurement to another unit of measurement.
<b>QB. Quantities can be added and subtracted only when they are of the same general type (length, area, speed, etc.).</b>	<b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; <u>apply such units correctly in expressions, equations and problem solutions that involve measurements</u> ; and convert a measurement using one unit of measurement to another unit of measurement.
<b>QC. Quantities can be multiplied or divided to create new types of quantities, called derived quantities.</b>	<b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; <u>apply such units correctly in expressions, equations and problem solutions that involve measurements</u> ; and convert a measurement using one unit of measurement to another unit of measurement.
<b>Core Skills</b>	
<b>Q1. Know when and how to convert units in computations.</b> Include the addition and subtraction of quantities of the same type expressed in different units; averaging data given in mixed units; converting units for derived quantities such as density and speed.	<b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; <u>apply such units correctly in expressions, equations and problem solutions that involve measurements</u> ; and <u>convert a measurement using one unit of measurement to another unit of measurement</u> .
<b>Q2. Use and interpret quantities and units correctly in algebraic formulas.</b> Include specifying units when defining variables and attending to units when writing expressions and equations.	<b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; <u>apply such units correctly in expressions, equations and problem solutions that involve measurements</u> ; and convert a measurement using one unit of

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<p><b>Q3. Use and interpret quantities and units correctly in graphs and data displays.</b>            Include function graphs, data tables, scatter plots and other visual displays of dimensioned data.</p>	<p>measurement to another unit of measurement.</p> <p><b>K8.1.</b> Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; <u>apply such units correctly in expressions, equations and problem solutions that involve measurements</u>; and convert a measurement using one unit of measurement to another unit of measurement.</p> <p><b>L1.1</b> <u>Organize and display data using appropriate methods</u> (including spreadsheets) to detect patterns and departures from patterns.</p>
<p><b>Q4. Use units as a way to understand problems and to guide the solution of multi-step problems.</b>            Include examples such as acceleration; currency conversions; people-hours; social science measures, such as deaths per 100,000; and general rate, such as points per game.</p>	<p><b>MR8.</b> When solving problems, <u>thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions</u> as a regular part of routine work, and devising independent ways to verify results.</p>
<p><b>Expressions</b>            Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.</p> <p>Reading an expression with comprehension involves analysis of its underlying structure, which may suggest a different but equivalent way of writing it that exhibits some different aspect of its meaning. For example, <math>p + 0.05p</math> can be interpreted as the addition of a 5% tax to a price <math>p</math>. But rewriting <math>p + 0.05p</math> as <math>1.05p</math> shows that adding a tax is the same as multiplying by a constant factor.</p> <p>Algebraic manipulations are based on the conventions of algebraic notation and the rules of arithmetic. Heuristic mnemonic devices are not a substitute for procedural fluency. For example, factoring, expanding, collecting like terms, the rules for interpreting minus signs next to</p>	

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<p>parenthetical sums, and adding fractions with a common denominator are all instances of the distributive law; the definitions for negative and rational exponents are based on the extension of the exponent laws for positive integers. The laws of exponents connect multiplication of numbers to addition of exponents and thus express the deep relationship between addition and multiplication captured by the parallel nature of the rules of arithmetic for these operations.</p> <p>Complex expressions are made up of simpler expressions using arithmetic operations and substitution. When simple expressions within more complex expressions are treated as single quantities, or chunks, the underlying structure of the larger expression may be more evident.</p> <p><i>Connections to Equations and Functions.</i> Setting expressions equal to each other leads to equations. Expressions can define functions of the variables that appear in them, with equivalent expressions defining the same function.</p>	
<b>Core Concepts</b>	
<b>ExA. Expressions are constructions built up from numbers, variables, and operations, which have a numerical value when each variable is replaced with a number.</b>	<p><i>While no specific ADP Benchmarks address these Core Concepts, the Algebra benchmarks presume this knowledge in the J1 expectations that call for performing basic operations on algebraic expressions.</i></p>
<b>ExB. Complex expressions are made up of simpler expressions.</b>	
<b>ExC. The rules of arithmetic can be applied to transform an expression without changing its value.</b>	
<b>ExD. Rewriting expressions serves a purpose in solving problems.</b>	<p><b>MR8.</b> <u>When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results.</u></p>
<b>Core Skills</b>	
<b>Ex1. See structure in expressions.</b>	<b>MR9.</b> Shifting regularly between the specific and the general,

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<p>For example, recognize: that the expressions <math>x^4 - y^4</math> and <math>(x + y)^2 - (x - y)^2</math> are differences of squares; that there are different ways to rewrite the latter expression, e.g., by expanding and collecting like terms or by factoring as a difference of squares; that <math>p</math> is a common factor in <math>p + 0.025p</math>; that an expression in the form <math>(x - 3)^2 + 14</math> reveals its minimum value.</p>	<p>using examples to understand general ideas, and extending specific results to more general cases to gain insight.</p>
<p><b>Ex2. Manipulate simple expressions.</b> Show procedural fluency in the following cases: factoring out common terms; factoring expressions with quadratic structure; writing in standard form sums, differences, and products of polynomials. Include completing the square and rewriting in standard form sums, differences, products, and quotients of simple rational expressions; rewriting expressions with negative exponents and those involving square or cube roots of a single term involving exponents.</p>	<p><b>J1.</b> Perform basic operations on algebraic expressions fluently and accurately:</p> <p><b>J1.1.</b> Understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions.</p> <p><b>J1.2.</b> * Understand the properties of rational exponents and apply these properties to simplify algebraic expressions.</p> <p><b>J1.3.</b> Add, subtract and multiply polynomials; divide a polynomial by a low degree polynomial.</p> <p><b>J1.4.</b> Factor polynomials by removing the greatest common factor; factor quadratic polynomials.</p> <p><b>J1.5.</b> Add, subtract, multiply, divide and simplify rational expressions.</p>
<p><b>Ex3. Define variables and write an expression to represent a quantity in a problem.</b> Include contextual problems.</p>	<p><b>MR7.</b> Recognizing and using the process of mathematical modeling: <u>recognizing and clarifying mathematical structures</u> that are embedded in other contexts, <u>formulating a problem in mathematical terms</u>, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<p><b>Ex4. Interpret an expression that represents a quantity in terms of the context.</b> Include interpreting parts of an expression, such as terms, factors and coefficients.</p>	<p><b>MR2.</b> Using multiple representations (literal, symbolic, graphic) to represent problems and solutions.</p> <p><b>MR7.</b> Recognizing and using the process of mathematical modeling: <u>recognizing and clarifying mathematical structures</u></p>

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	<p><u>that are embedded in other contexts</u>, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<p><b>Equations</b>            An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true. If the equation is true for all values of the variables, then we call it an identity; identities are often discovered by manipulating one expression into another.</p> <p>The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs, which can be graphed in the plane. Equations can be combined into systems to be solved simultaneously.</p> <p>An equation can be solved by successively transforming it into one or more simpler equations. The process is governed by deductions based on the properties of equality. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.</p> <p>Some equations have no solutions in a given number system, stimulating the formation of expanded number systems (integers, rational numbers, real numbers and complex numbers).</p> <p>A formula is a type of equation. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, <math>A = ((b_1 + b_2)/2) * h</math>, can be solved for <math>h</math> using the same deductive process.</p>	

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<p>Inequalities can be solved in much the same way as equations. Many, but not all, of the properties of equality extend to the solution of inequalities.</p> <p><i>Connections to Functions, Coordinates, and Modeling.</i> Equations in two variables may define functions. Asking when two functions have the same value leads to an equation; graphing the two functions allows for the approximate solution of the equation. Equations of lines involve coordinates, and converting verbal descriptions to equations is an essential skill in modeling.</p>	
<b>Core Concepts</b>	
<b>EqA. An equation is a statement that two expressions are equal.</b>	<p><i>While no specific ADP Benchmarks articulate these concepts, they are assumed in the Algebra benchmarks (J3) that call for students to use basic algebraic operations to solve equations.</i></p>
<b>EqB. The solutions of an equation are the values of the variables that make the resulting numerical statement true.</b>	
<b>EqC. The steps in solving an equation are guided by understanding and justified by logical reasoning.</b>	
<b>EqD. Equations not solvable in one number system may have solutions in a larger system.</b>	
<b>Core Skills</b>	
<p><b>Eq1. Understand a problem and formulate an equation to solve it.</b> Extend to inequalities and systems.</p>	<p><b>J5. <u>Solve problems by converting the verbal information given into an appropriate mathematical model involving equations or systems of equations;</u></b> apply appropriate mathematical techniques to analyze these mathematical models; and interpret the solution obtained in written form using appropriate units of measurement:</p>
<b>Eq2. Solve equations in one variable using manipulations guided</b>	<b>I2.1</b> Locate the position of a number on the number line, <u>know</u>

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<p><b>by the rules of arithmetic and the properties of equality.</b> Solve linear equations with procedural fluency. For quadratic equations, include solution by inspection, by factoring, or by using the quadratic formula. Understand that the quadratic formula comes from completing the square. Include simple absolute value equations solvable by direct inspection and by understanding the interpretation of absolute value as distance.</p>	<p><u>that its distance from the origin is its absolute value and know that the distance between two numbers on the number line is the absolute value of their difference.</u></p> <p><b>J3.1.</b> Solve linear equations and inequalities in one variable including those involving the absolute value of a linear function.</p> <p><b>J3.5.</b> Solve quadratic equations in one variable.</p>
<p><b>Eq3. Rearrange formulas to isolate a quantity of interest.</b> Exclude cases that require extraction of roots or inverse functions.</p>	<p><b>J3.2.</b> Solve an equation involving several variables for one variable in terms of the others.</p>
<p><b>Eq4. Solve systems of equations.</b> Focus on pairs of simultaneous linear equations in two variables. Include algebraic techniques, graphical techniques and solving by inspection.</p>	<p><b>J3.3.</b> Solve systems of two linear equations in two variables.</p> <p><b>J4.3.</b> Understand the relationship between a solution of a system of two linear equations in two variables and the graphs of the corresponding lines.</p> <p><b>MR2.</b> Using multiple representations (literal, symbolic, graphic) to represent problems and solutions.</p>
<p><b>Eq5. Solve linear inequalities in one variable and graph the solution set on a number line.</b> Emphasize solving the associated equality and determining on which side of the solution of the associated equation the solutions to the inequality lie.</p>	<p><b>J3.1.</b> Solve linear equations and <u>inequalities</u> in one variable including those involving the absolute value of a linear function.</p>
<p><b>Eq6. Graph the solution set of a linear inequality in two variables on the coordinate plane.</b> Emphasize graphing the associated equation, using a dashed or solid line as appropriate and shading to indicate the half-plane on which the solutions to the inequality lie.</p>	<p><b>J3.1.</b> Solve linear equations and <u>inequalities</u> in one variable including those involving the absolute value of a linear function.</p> <p><b>J4.4.</b> <u>Graph the solution set of a linear inequality and identify whether the solution set is an open or a closed half-plane;</u> graph the solution set of a system of two or three linear inequalities.</p>
<p><b>Functions</b> Functions model situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage</p>	



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<p>rate of 4.25% is a function of the length of time the money is invested. Because nature and society are full of dependencies between quantities, functions are important tools in the construction of mathematical models.</p> <p>In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a plane to fly 1000 miles is a function of the plane's average ground speed in miles per hour, <math>v</math>; the rule <math>T(v) = 1000/v</math> expresses this relationship algebraically and defines a function whose name is <math>T</math>.</p> <p>The set of possible inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. The graph of a function is a useful way of visualizing the relationship the function models, and manipulating the expression for a function can throw light on the function's properties.</p> <p>Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with an initial value of zero describe proportional relationships.</p> <p><i>Connections to Expressions, Equations, Modeling and Coordinates.</i> Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. The graph of a function <math>f</math> is the same as the solution set of the equation <math>y = f(x)</math>. Questions about when two functions have the same value lead to equations, whose solutions can be visualized from the intersection of the graphs. Since functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be modeled effectively using a spreadsheet or other technology.</p>	

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<b>Core Concepts</b>	
<b>FA. A function is a rule, often defined by an expression, that assigns a unique output for every input.</b>	<b>J2.1</b> Recognize whether a relationship given in symbolic or graphical form is a function
<b>FB. The graph of a function <math>f</math> is a set of ordered pairs <math>(x, f(x))</math> in the coordinate plane.</b>	<b>K10.</b> <u>Represent geometric objects and figures algebraically using coordinates</u> ; use algebra to solve geometric problems: <b>J2.2.</b> * Determine the domain of a function represented in either symbolic or graphical form. <b>J2.3.</b> Understand functional notation and evaluate a function at a specified point in its domain.
<b>FC. Functions model situations where one quantity determines another.</b>	<b>J2.1</b> Recognize whether a relationship given in symbolic or graphical form is a function <b>J5.4.</b> <u>Recognize and solve problems that can be modeled using an exponential function</u> , such as compound interest problems.
<b>FD. Common functions occur in families where each member describes a similar type of dependence.</b>	
<b>Core Skills</b>	
<b>F1. Recognize proportional relationships and solve problems involving rates and ratios.</b> Include being able to express proportional relationships as functions.	<b>I1.2.</b> Calculate and apply <u>ratios, proportions, rates</u> and percentages to solve problems. <b>J5.1.</b> Recognize and solve problems that can be modeled using a linear equation in one variable, such as <u>time/rate/distance</u> problems, percentage increase or decrease problems, and <u>ratio and proportion</u> problems.
<b>F2. Describe the qualitative behavior of common types of functions using graphs and tables.</b> Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Use technology to explore the effects of parameter changes on the graphs of linear, power, quadratic, polynomial, simple rational, exponential, logarithmic, sine and	<b>MR2.</b> Using multiple representations (literal, symbolic, graphic) to represent problems and solutions. <b>J1.1.</b> Understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions. <b>J1.6.</b> Evaluate <u>polynomial and rational expressions</u> and

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<p>cosine, absolute value and step functions.</p>	<p>expressions containing radicals and <u>absolute values</u> at specified values of their variables.</p> <p><b>J4.8.</b> Read information and draw conclusions from graphs; identify properties of a graph that provide useful information about the original problem.</p> <p><b>J4.2.</b> Understand the relationship between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph.</p> <p><b>J4.5.</b> Graph a quadratic function and understand the relationship between its real zeros and the x-intercepts of its graph.</p> <p><b>J4.7.</b> Graph exponential functions and identify their key characteristics.</p> <p><b>K12.3.</b> * Graph sine, cosine and tangent as well as their reciprocals, secant, cosecant and cotangent; identify key characteristics.</p>
<p><b>F3. Analyze functions using symbolic manipulation.</b>            Include slope-intercept and point-slope form of linear functions; factored form to find horizontal intercepts; vertex form of quadratic functions to identify symmetry and find maximums and minimums; factored form to find zeros. Use manipulations as described under Expressions.</p>	<p><b>K10.2.</b> Describe a line by a linear equation.</p> <p><b>J4.2.</b> Understand the relationship between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph.</p> <p><b>J4.5.</b> Graph a quadratic function and understand the relationship between its real zeros and the x-intercepts of its graph.</p>
<p><b>F4. Use the families of linear and exponential functions to solve problems.</b>            For linear functions <math>f(x) = mx + b</math>, understand <math>b</math> as the intercept or initial value and <math>m</math> as the slope or rate of change. For exponential functions <math>f(x) = a \cdot b^x</math>, understand <math>a</math> as the intercept or initial value and <math>b</math> as the growth factor.</p>	<p><b>J5.1.</b> Recognize and <u>solve problems that can be modeled using a linear equation</u> in one variable, such as time/rate/distance problems, percentage increase or decrease problems, and ratio and proportion problems.</p> <p><b>J5.4.</b> Recognize and <u>solve problems that can be modeled using an exponential function</u>, such as compound interest problems.</p>

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<p><b>F5. Find and interpret rates of change.</b>            Compute the rate of change of linear functions and make qualitative observations about how the rate of change varies for nonlinear functions.</p>	<p><b>J4.1.</b> Graph a linear equation and demonstrate that it has a constant rate of change.</p> <p><b>K10.1.</b> Express the intuitive concept of the “slant” of a line in terms of the precise concept of slope, use the coordinates of two points on a line to define its slope, and use slope to express the parallelism and perpendicularity of lines.</p>
<p><b>Modeling</b>            Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.</p> <p>A model can be very simple, such as a geometric shape to describe a physical object like a coin. Even so simple a model involves making choices. It is up to us whether to model the solid nature of the coin with a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. For some purposes, we might even choose to adjust the right circular cylinder to model more closely the way the coin deviates from the cylinder.</p> <p>In any given situation, the model we devise depends on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models we can create and analyze is constrained as well by the limitations of our mathematical and technical skills. For example, modeling a physical object, a delivery route, a production schedule, or a comparison of loan amortizations each requires different sets of tools. Networks, spreadsheets and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical structure might model seemingly different situations.</p> <p>The basic modeling cycle is one of (1) identifying the key features of a</p>	

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<p>situation, (2) creating geometric, algebraic or statistical objects that describe key features of the situation, (3) analyzing and performing operations on these objects to draw conclusions and (4) interpreting the results of the mathematics in terms of the original situation. Choices and assumptions are present throughout this cycle.</p> <p><i>Connections to Quantity, Equations, Functions, Shape, Coordinates and Statistics.</i> Modeling makes use of shape, data, graphs, equations and functions to represent real-world quantities and situations.</p>	
<b>Core Concepts</b>	
<p><b>MA. Mathematical models involve choices and assumptions that abstract key features from situations to help us solve problems.</b></p>	<p><b>MR7.</b> Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p> <p><b>J5.</b> Solve problems by converting the verbal information given into an appropriate mathematical model involving equations or systems of equations; apply appropriate mathematical techniques to analyze these mathematical models; and interpret the solution obtained in written form using appropriate units of measurement.</p>
<p><b>MB. Even very simple models can be useful.</b></p>	<p><b>MR7.</b> Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<b>Core Skills</b>	
<p><b>M1. Model numerical situations.</b> Include readily applying the four basic operations in combination to solve</p>	<p><b>I1.1.</b> Add, subtract, multiply and divide integers, fractions and decimals.</p>

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<p>multi-step quantitative problems with dimensioned quantities; making estimates to introduce numbers into a situation and get problems started; recognizing proportional or near-proportional relationships and analyzing them using characteristic rates and ratios.</p>	<p><b>I1.2.</b> Calculate and apply ratios, proportions, rates and percentages to solve problems.</p> <p><b>MR5.</b> Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations.</p> <p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling:</u> recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<p><b>M2. Model physical objects with geometric shapes.</b> Include common objects that can reasonably be idealized as two- and three-dimensional geometric shapes. Identify the ways in which the actual shape varies from the idealized geometric model.</p>	<p><b>K9.</b> Visualize solids and surfaces in three-dimensional space when given two-dimensional representations (e.g., nets, multiple views) and create two-dimensional representations for the surfaces of three-dimensional objects.</p> <p><b>K6.</b> Use rigid motions (compositions of reflections, translations and rotations) to determine whether two geometric figures are congruent and to <u>create and analyze geometric designs.</u></p> <p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling:</u> recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<p><b>M3. Model situations with equations and inequalities.</b> Include situations well described by a linear inequality in two variables or a system of linear inequalities defining a region in the plane.</p>	<p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling:</u> recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>

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	<p><b>J5.1.</b> Recognize and solve problems that can be <u>modeled using a linear equation</u> in one variable, such as time/rate/distance problems, percentage increase or decrease problems, and ratio and proportion problems.</p> <p><b>J5.2.</b> Recognize and solve problems that can be <u>modeled using a system of two equations</u> in two variables, such as mixture problems.</p> <p><b>J5.3.</b> Recognize and solve problems that can be <u>modeled using a quadratic equation</u>, such as the motion of an object under the force of gravity.</p> <p><b>J5.6.</b> Recognize and solve problems that can be <u>modeled using a finite geometric series</u>, such as home mortgage problems and other compound interest problems.</p>
<p><b>M4. Model situations with common functions.</b>            Include situations well described by linear, quadratic or exponential functions; and situations that can be well described by inverse variation (<math>y = \frac{k}{x}</math>). Include identifying a family of functions that models features of a problem, and identifying a particular function of that family and adjusting it to fit by changing parameters. Understand the recursive nature of situations modeled by linear and exponential functions.</p>	<p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling</u>: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p> <p><b>J5.1.</b> Recognize and solve problems that can be <u>modeled using a linear equation</u> in one variable, such as time/rate/distance problems, percentage increase or decrease problems, and ratio and proportion problems.</p> <p><b>J5.3.</b> Recognize and solve problems that can be <u>modeled using a quadratic equation</u>, such as the motion of an object under the force of gravity.</p> <p><b>J5.4.</b> Recognize and solve problems that can be <u>modeled using an exponential function</u>, such as compound interest problems.</p> <p><b>J5.6.</b> Recognize and solve problems that can be modeled using a finite geometric series, such as home mortgage</p>

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<p><b>M5. Model situations using probability and statistics.</b>            Include using simulations to model probabilistic situations; describing the shape of a distribution of values and summarizing a distribution with measures of center and variability; modeling a bivariate relationship using a trend line or a regression line.</p>	<p>problems and other compound interest problems.</p> <p><b>L1.3.</b> Compute and explain summary statistics for distributions of data including <u>measures of center</u> (mean, median) and spread (range, percentiles, variance, standard deviation).</p> <p><b>L1.5.</b> Create scatter plots, analyze patterns and describe relationships in paired data.</p> <p><b>L1.6.</b> Know the characteristics of the Gaussian normal distribution (bell-shaped curve).</p> <p><b>L3.4.</b> Construct a scatter plot of a set of paired data, and if it demonstrates a <u>linear trend</u>, use a graphing calculator to find the <u>regression line</u> that best fits this data; recognize that the correlation coefficient measures goodness of fit and explain when it is appropriate to use the regression line to make predictions.</p> <p><b>L4.2.</b> Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome.</p> <p><b>L4.3.</b> Explain how the law of large numbers can be applied in simple examples.</p> <p><b>L4.5.</b> Apply probability concepts to practical situations to make informed decisions.</p> <p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling</u>: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.</p>
<p><b>M6. Interpret the results of applying a model and compare models for a particular situation.</b></p>	<p><b>MR7.</b> Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures</p>



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<p>Include realizing that models seldom fit exactly and so there can be error; identifying simple sources of error and being careful not to over-interpret models. Include recognizing that there can be many models that relate to a situation, that they can capture different aspects of the situation, that they can be simpler or more complex, and that they can have a better or worse fit to the situation and the questions being asked.</p>	<p>that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and <u>interpreting the solution in the context of the original problem.</u></p>
<p><b>Shape</b>            From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships. Once understood, those attributes and relationships can be applied in diverse practical situations—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.</p> <p>Understanding the attributes of geometric objects often relies on measurement: a circle is a set of points in a plane at a fixed distance from a point; a cube is bounded by six squares of equal area; when two parallel lines are crossed by a transversal, pairs of corresponding angles are congruent.</p> <p>The concepts of congruence, similarity and symmetry can be united under the concept of geometric transformation. Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. Applying a scale transformation to a geometric figure yields a similar figure. The transformation preserves angle measure, and lengths are related by a constant of proportionality. If the constant of proportionality is one, distances are also preserved (so the transformation is a rigid transformation) and the figures are congruent.</p> <p>The definitions of sine, cosine and tangent for acute angles are founded</p>	

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<p>on right triangle similarity, and, with the Pythagorean theorem, are fundamental in many practical and theoretical situations.</p> <p><i>Connections to Coordinates, Functions and Modeling.</i> The Pythagorean theorem is a key link between geometry, measurement and distance in the coordinate plane. Parameter changes in families of functions can be interpreted as transformations applied to their graphs and those functions, as well as geometric objects in their own right, can be used to model contextual situations.</p>	
<b>Core Concepts</b>	
<p><b>SpA. Shapes and their parts, attributes, and their measurements can be analyzed deductively.</b></p>	<p><b>K2.1.</b> Identify and apply properties of and theorems about <u>parallel lines</u> and use them to <u>prove theorems</u> such as two lines parallel to a third are parallel to each other and to perform constructions such as a line parallel to a given line through a point not on the line.</p>
	<p><b>K2.2.</b> Identify and apply properties of and theorems about <u>perpendicular lines</u> and use them to <u>prove theorems</u> such as the perpendicular bisectors of line segments are the set of all points equidistant from the two end points and to perform constructions such as the perpendicular bisector of a line segment.</p>
	<p><b>K2.3.</b> Identify and apply properties of and theorems about <u>angles</u> and use them to <u>prove theorems</u> such as two lines are parallel exactly when the alternate interior angles they make with a transversal are equal and to perform constructions such as the bisector of an angle.</p>
	<p><b>K3.</b> Know the basic theorems about <u>congruent and similar triangles</u> and use them to <u>prove additional theorems</u> and solve problems.</p>
	<p><b>K4.</b> Know the definitions and basic properties of a <u>circle</u> and use them to <u>prove basic theorems</u> and solve problems.</p>



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<p><b>SpB. Congruence, similarity, and symmetry can be analyzed using transformations.</b></p>	<p><b>K6.</b> Use rigid motions (compositions of reflections, translations and rotations) to determine whether two geometric figures are congruent and to create and analyze geometric designs.</p>
<p><b>SpC. Mathematical shapes model the physical world, resulting in practical applications of geometry.</b></p>	<p><b>K9.</b> Visualize solids and surfaces in three-dimensional space when given two-dimensional representations (e.g., nets, multiple views) and create two-dimensional representations for the surfaces of three-dimensional objects.</p>
	<p><b>MR7.</b> <u>Recognizing and using the process of mathematical modeling</u>: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem</p>
<p><b>SpD. Right triangles and the Pythagorean theorem are central to geometry and its applications, including trigonometry.</b></p>	<p><b>K11.</b> Understand basic right-triangle trigonometry and apply it to solve problems:</p>
	<p><b>K11.1.</b> Understand how similarity of right triangles allows the trigonometric functions sine, cosine and tangent to be defined as ratios of sides and be able to use these functions to solve problems.</p>
	<p><b>K11.2.</b> Apply the trigonometric functions sine, cosine and tangent to solve for an unknown length of a side of a right triangle, given one of the acute angles and the length of another side.</p>
	<p><b>K1.2.</b> State and prove key <u>basic theorems in geometry such as the Pythagorean theorem</u>, the sum of the angles of a triangle is 180 degrees, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.</p>
	<p><b>K5.</b> <u>Apply the Pythagorean theorem</u>, its converse and</p>

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<b>Core Skills</b>	properties of special right triangles to solve problems.
<p><b>Sp1. Use multiple geometric properties to solve problems involving geometric figures.</b>            Properties include: measures of interior angles of a triangle sum to <math>180^\circ</math>; vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; measures of supplementary angles sum to <math>180^\circ</math>; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints; and a line tangent to a circle is perpendicular to the radius meeting it.</p>	<p><b>K2.</b> Identify and <u>apply</u> the definitions related to <u>lines and angles and use them</u> to prove theorems in (Euclidean) geometry, <u>solve problems</u>, and perform basic geometric constructions using a straight edge and compass:</p> <p><b>K2.1.</b> Identify and <u>apply properties of</u> and theorems about <u>parallel lines</u> and use them to prove theorems such as two lines parallel to a third are parallel to each other and to perform constructions such as a line parallel to a given line through a point not on the line.</p> <p><b>K2.2.</b> Identify and <u>apply properties of</u> and theorems about <u>perpendicular lines</u> and use them to prove theorems such as the perpendicular bisectors of line segments are the set of all points equidistant from the two end points and to perform constructions such as the perpendicular bisector of a line segment.</p> <p><b>K2.3.</b> Identify and <u>apply properties of</u> and theorems about <u>angles</u> and use them to prove theorems such as two lines are parallel exactly when the alternate interior angles they make with a transversal are equal and to perform constructions such as the bisector of an angle.</p> <p><b>K4.</b> Know the definitions and basic <u>properties of a circle</u> and use them to prove basic theorems and <u>solve problems</u>.</p>
<p><b>Sp2. Prove theorems, test conjectures and identify logical errors.</b>            Include theorems establishing the properties in Core Skill 1 and other theorems about angles, parallel and perpendicular lines, similarity and congruence of triangles.</p>	<p><b>MR8.</b> When solving problems, thinking ahead about strategy, <u>testing ideas with special cases</u>, trying different approaches, <u>checking for errors</u> and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results.</p> <p><b>MR1.</b> Using <u>inductive and deductive reasoning</u> to arrive at valid conclusions.</p>

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	<b>MR3.</b> Understanding the role of definitions, proofs and counterexamples in mathematical reasoning; <u>constructing simple proofs</u> .
	<b>K1.1.</b> Identify, explain the necessity of and give examples of definitions, axioms and theorems.
	<b>K1.2.</b> State and prove key <u>basic theorems in geometry such as the Pythagorean theorem</u> , the sum of the angles of a triangle is 180 degrees, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
	<b>K2.1.</b> Identify and apply properties of and theorems about <u>parallel lines</u> and use them to <u>prove theorems</u> such as two lines parallel to a third are parallel to each other and to perform constructions such as a line parallel to a given line through a point not on the line.
	<b>K2.2.</b> Identify and apply properties of and theorems about <u>perpendicular lines</u> and use them to <u>prove theorems</u> such as the perpendicular bisectors of line segments are the set of all points equidistant from the two end points and to perform constructions such as the perpendicular bisector of a line segment.
	<b>K2.3.</b> Identify and apply properties of and theorems about <u>angles</u> and use them to <u>prove theorems</u> such as two lines are parallel exactly when the alternate interior angles they make with a transversal are equal and to perform constructions such as the bisector of an angle.
	<b>K3.</b> Know the basic theorems about <u>congruent and similar triangles</u> and use them to <u>prove additional theorems</u> and solve problems.
	<b>K4.</b> Know the definitions and basic properties of a circle and use them to <u>prove basic theorems</u> and solve problems.

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<p><b>Sp3. Construct and interpret representations of geometric objects.</b>            Include classical construction techniques and construction techniques supported by modern technologies. Include moving between two-dimensional representations and the three-dimensional objects they represent, such as in schematics, assembly instructions, perspective drawings and multiple views.</p>	<p><b>K2.</b> Identify and apply the definitions related to lines and angles and use them to prove theorems in (Euclidean) geometry, solve problems, and <u>perform basic geometric constructions using a straight edge and compass</u>:</p>
	<p><b>K2.1.</b> Identify and apply properties of and theorems about parallel lines and use them to prove theorems such as two lines parallel to a third are parallel to each other and to <u>perform constructions</u> such as a line parallel to a given line through a point not on the line.</p>
	<p><b>K2.2.</b> Identify and apply properties of and theorems about perpendicular lines and use them to prove theorems such as the perpendicular bisectors of line segments are the set of all points equidistant from the two end points and to <u>perform constructions</u> such as the perpendicular bisector of a line segment.</p>
	<p><b>K2.3.</b> Identify and apply properties of and theorems about angles and use them to prove theorems such as two lines are parallel exactly when the alternate interior angles they make with a transversal are equal and to <u>perform constructions</u> such as the bisector of an angle.</p>
	<p><b>K9.</b> Visualize solids and surfaces in three-dimensional space when given two-dimensional representations (e.g., nets, multiple views) and create two-dimensional representations for the surfaces of three-dimensional objects.</p>
<p><b>Sp4. Solve problems involving measurements.</b>            Include measurement (length, angle measure, area, surface area, and volume) of a variety of figures and shapes in two- and three-dimensions. Compute measurements using formulas and by decomposing complex shapes into simpler ones.</p>	<p><b>K8.2.</b> Determine the perimeter of a polygon and the circumference of a circle; the area of a rectangle, a circle, a triangle and a polygon with more than four sides by decomposing it into triangles; the surface area of a prism, a pyramid, a cone and a sphere; and the volume of a rectangular box, a prism, a pyramid, a cone and a sphere.</p>

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<p><b>Sp5. Solve problems involving similar triangles and scale drawings.</b>            Include computing actual lengths, areas and volumes from a scale drawing and reproducing a scale drawing at a different scale.</p>	<p><b>K3.</b> Know the basic theorems about congruent and <u>similar triangles</u> and use them to prove additional theorems and <u>solve problems</u>.</p> <p><b>K7.</b> Know about the <u>similarity of figures</u> and use the <u>scale factor</u> to solve problems.</p> <p><b>K8.3.</b> Know that <u>the effect of a scale factor k on length, area and volume</u> is to multiply each by k, <math>k^2</math> and <math>k^3</math>, respectively.</p>
<p><b>Sp6. Apply properties of right triangles and right triangle trigonometry to solve problems.</b>            Include using the Pythagorean theorem and properties of special right triangles, and applying sine, cosine and tangent to determine lengths and angle measures of right triangles. Use right triangles and their properties to solve real-world problems. Limit angle measures to degrees.</p>	<p><b>K11.1.</b> Understand how <u>similarity of right triangles</u> allows the trigonometric functions sine, cosine and tangent to be defined as ratios of sides and be able to use these functions to <u>solve problems</u>.</p> <p><b>K11.2.</b> <u>Apply the trigonometric functions</u> sine, cosine and tangent to solve for an unknown length of a side of a right triangle, given one of the acute angles and the length of another side.</p> <p><b>K5.</b> <u>Apply the Pythagorean theorem</u>, its converse and properties of special right triangles to solve problems.</p>
<p><b>Coordinates</b>            Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p> <p>Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling and proof.</p>	

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<p>Coordinate geometry is a rich field for exploration. How does a geometric transformation such as a translation or reflection affect the coordinates of points? How is the geometric definition of a circle reflected in its equation?</p> <p>Adding a third perpendicular axis associates three numbers with locations in three dimensions and extends the use of algebraic techniques to problems involving the three-dimensional world we live in.</p> <p><i>Connections to Shape, Quantity, Equations and Functions.</i> Coordinates can be used to reason about shapes. In applications, coordinate values often have units (such as meters and bushels). A one-variable equation of the form <math>f(x) = g(x)</math> may be solved in the coordinate plane by finding intersections of the curves <math>y = f(x)</math> and <math>y = g(x)</math>.</p>	
<b>Core Concepts</b>	
<b>CA. Locations in the plane or space can be specified by pairs or triples of numbers called coordinates.</b>	<b>K10.</b> Represent geometric objects and figures algebraically <u>using coordinates</u> ; use algebra to solve geometric problems:
<b>CB. Coordinates link algebra with geometry and allow methods in one domain to solve problems in the other.</b>	<b>K10.</b> <u>Represent geometric objects and figures algebraically using coordinates</u> ; use <u>algebra to solve geometric problems</u> :
<b>CC. The set of solutions to an equation in two variables forms a curve in the coordinate plane—such as a line, parabola, circle—and the solutions to systems of equations in two variables correspond to intersections of these curves.</b>	<b>J4.</b> <u>Graph a variety of equations and inequalities in two variables</u> , demonstrate understanding of <u>the relationships between the algebraic properties of an equation and the geometric properties of its graph</u> , and interpret a graph:
	<b>J4.1.</b> <u>Graph a linear equation</u> and demonstrate that it has a constant rate of change.
	<b>J4.5.</b> <u>Graph a quadratic function</u> and understand the relationship between its real zeros and the x-intercepts of its graph.
	<b>J4.7.</b> Graph exponential functions and identify their key characteristics.
	<b>J4.3.</b> Understand the relationship between a solution of a

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	<u>system of two linear equations</u> in two variables and the graphs of the corresponding lines.
<b>Core Skills</b>	
<b>C1. Translate fluently between lines in the coordinate plane and their equations.</b> Include predicting visual features of lines by inspection of their equations, determining the equation of the line through two given points, and determining the equation of the line with a given slope passing through a given point.	<b>J4.1.</b> Graph a linear equation and demonstrate that it has a constant rate of change.  <b>J4.2.</b> Understand the relationship between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph.  <b>K10.2.</b> Describe a line by a linear equation.
<b>C2. Identify the correspondence between parameters in common families of equations and the location and appearance of their graphs.</b> Include common families of equations—the graphs of $Ax + By = C$ , $y = mx + b$ and $x = a$ are straight lines; the graphs of $y = a(x - h)^2 + k$ and $y = Ax^2 + Bx + C$ are parabolas; and the graph of $(x - h)^2 + (y - k)^2 = r^2$ is a circle.	<b>J4.2.</b> <u>Understand the relationship</u> between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph.  <b>J4.5.</b> Graph a quadratic function and <u>understand the relationship</u> between its real zeros and the x-intercepts of its graph.  <b>K10.4.</b> * <u>Find an equation of a circle</u> given its center and radius and, given an equation of a circle, find its center and radius.
<b>C3. Use coordinates to solve geometric problems.</b> Include proving simple theorems algebraically, using coordinates to compute perimeters and areas for triangles and rectangles, finding midpoints of line segments, finding distances between pairs of points and determining when two lines are parallel or perpendicular.	<b>K10.</b> <u>Represent geometric objects</u> and figures algebraically using coordinates; use algebra <u>to solve geometric problems</u> :  <b>K10.1.</b> Express the intuitive concept of the “slant” of a line in terms of the precise concept of slope, use the coordinates of two points on a line to define its slope, and <u>use slope to express the parallelism and perpendicularity of lines</u> .  <b>K10.3.</b> Find the <u>distance between two points</u> using their coordinates and the Pythagorean theorem.
<b>Probability</b> Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1. This number is generally	



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<p>interpreted as the relative frequency of occurrence of the event over the long run.</p> <p>The structure of a probability model begins by listing or describing the possible outcomes for a random situation (the sample space) and assigning probabilities based on an assumption about long-run relative frequency. In situations such as flipping a coin, rolling a number cube, or drawing a card, it is reasonable to assume various outcomes are equally likely.</p> <p>Compound events constructed from these simple ones can be represented by tree diagrams and by frequency or relative frequency tables. The probabilities of compound events can be computed using these representations and by applying the additive and multiplicative laws of probability. Interpreting these probabilities relies on an understanding of independence and conditional probability, approachable through the analysis of two-way tables.</p> <p>Converting a verbally-stated problem into the symbols and relations of probability requires careful attention to words such as <i>and</i>, <i>or</i>, <i>if</i>, and <i>all</i>, and to grammatical constructions that reflect logical connections. This is especially true when applying probability models to real-world problems, where simplifying assumptions are also usually necessary in order to gain at least an approximate solution.</p> <p><i>Connections to Statistics and Expressions.</i> Probability is the foundation for drawing valid conclusions from sampling or experimental data. Counting has an advanced connection with Expressions through Pascal's triangle and binomial expansions.</p>	
<b>Core Concepts</b>	
<p><b>PA. Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of uncertainty in terms of relative frequency of occurrence.</b></p>	<p><b>L4.1.</b> Explain how probability quantifies the likelihood that an event occurs in terms of numbers.</p>

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<b>PB. The law of large numbers provides the basis for estimating certain probabilities by use of empirical relative frequencies.</b>	<b>L4.1.</b> Explain how probability quantifies the likelihood that an event occurs in terms of numbers.
	<b>L4.2.</b> Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome.
	<b>L4.3.</b> Explain how the law of large numbers can be applied in simple examples.
<b>PC. The laws of probability govern the calculation of probabilities of combined events.</b>	<b>L4.4.</b> Apply probability concepts such as conditional probability and independent events to calculate simple probabilities.
<b>PD. Interpreting probabilities contextually is essential to rational decision-making in situations involving randomness.</b>	<b>L4.5.</b> Apply probability concepts to practical situations to <u>make informed decisions</u> .
<b>Core Skills</b>	
<b>P1. Compute theoretical probabilities by systematically counting points in the sample space.</b> Make use of symmetry and equally likely outcomes. Include permutation and combination problems as long as small numbers are involved or technology is used, so that formulas are not required.	<b>L4.1.</b> Explain how probability quantifies the likelihood that an event occurs in terms of numbers.
<b>P2. Interpret probabilities of compound events using concepts of independence and conditional probability.</b> Include reading conditional probabilities from two-way tables.	<b>L4.4.</b> Apply probability concepts such as conditional probability and independent events to calculate simple probabilities.
<b>P3. Compute probabilities of compound events.</b> Make use of the additive and multiplicative laws of probability, tree diagrams and frequency or relative frequency tables in real contexts. Do not emphasize fluency with the related formulas.	<b>L4.2.</b> Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome.
	<b>L4.4.</b> Apply probability concepts such as conditional probability and independent events to calculate simple probabilities.
	<b>L4.5.</b> Apply probability concepts to practical situations to make informed decisions.
<b>P4. Estimate probabilities empirically.</b>	<b>L4.2.</b> Explain how the relative frequency of a specified

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Include using data from simulations carried out with technology to estimate probabilities.	outcome of an event can be used to estimate the probability of the outcome.
<p><b>P5. Identify and explain common misconceptions regarding probability.</b></p> <p>Include misconceptions about long-run versus short-run behavior of relative frequencies (the law of large numbers). Include attention to the use and misuse of probability in the media, especially in terms of interpreting charts and tables and in the contextual meaning of terms connected to probability, such as ‘odds’ or ‘risk.’</p>	<p><b>L4.3.</b> Explain how the law of large numbers can be applied in simple examples.</p>
<p><b>P6. Adapt probability models to solve real-world problems.</b></p> <p>Include the use of conditional probability to assess subsets of data (e.g., what does the data say about males and females separately). Include the use of independence as a simplifying assumption (e.g., find the probability that two students both contract the disease this year).</p>	<p><b>L4.4.</b> Apply probability concepts such as <u>conditional probability and independent events</u> to calculate simple probabilities.</p> <p><b>L4.5.</b> Apply probability concepts to practical situations to make informed decisions.</p>
<p><b>Statistics</b></p> <p>Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.</p> <p>Data are gathered, displayed, summarized, examined and interpreted to discover patterns. Data can be summarized by a statistic measuring center, such as mean or median, and a statistic measuring spread, such as interquartile range or standard deviation. Different distributions can be compared numerically using these statistics or visually using plots. Which statistics to compare, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.</p> <p>Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population</p>	

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<p>makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance and this can be evaluated only under the condition of randomness.</p> <p>In critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were collected, and the analyses employed as well as the data summaries and the conclusions drawn.</p> <p><i>Connections to Probability, Functions and Modeling.</i> Valid conclusions about a population depend on designed simulations or other statistical studies using random sampling or assignment and rely on probability for their interpretation. Functional models may be used to approximate data. If the data are approximately linear, the relationship may be modeled with a trend line and the strength and direction of such a relationship may be expressed through a correlation coefficient. Technology facilitates the study of statistics by making it possible to simulate many possible outcomes in a short amount of time, and by generating plots, function models, trend lines and correlation coefficients.</p>	
<b>Core Concepts</b>	
<p><b>StA. Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.</b></p>	<p><b>L1.3.</b> Compute and explain summary statistics for distributions of data including measures of center (mean, median) and spread (range, percentiles, variance, standard deviation).</p> <p><b>L2.1.</b> Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed.</p>
<p><b>StB. Visual displays and summary statistics condense the information in data sets into usable knowledge.</b></p>	<p><b>L1.1.</b> Organize and display data using appropriate methods (including spreadsheets) to detect patterns and departures</p>

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	<p>from patterns.</p> <p><b>L1.3.</b> Compute and explain summary statistics for distributions of data including measures of center (mean, median) and spread (range, percentiles, variance, standard deviation).</p>
<p><b>StC. Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic.</b></p>	<p><b>L3.3.</b> Explain the differences between randomized experiments and observational studies.</p>
<p><b>StD. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.</b></p>	<p><b>L3.1.</b> Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection and the conclusions that can rightfully be made.</p>
<p><b>Core Skills</b></p>	
<p><b>St1. Formulate questions that can be addressed with data. Identify the relevant data, collect and organize it to respond to the question.</b>            Include determining whether a question can best be addressed through a sample survey, randomized experiment or observational study. Include unbiased selection for a sample and randomization of assignment to treatment for an experiment.</p>	<p><b>L3.1.</b> Explain the impact of sampling methods, <u>bias</u> and the <u>phrasing of questions asked during data collection</u> and the <u>conclusions that can rightfully be made</u>.</p> <p><b>L3.2.</b> Design simple experiments or investigations to <u>collect data to answer questions of interest</u>.</p> <p><b>L3.3.</b> Explain the differences between randomized experiments and observational studies.</p>
<p><b>St2. Use appropriate displays and summary statistics for data.</b>            Include univariate, bivariate, categorical and quantitative data. Include the thoughtful selection of measures of center and spread to summarize data.</p>	<p><b>L1.1.</b> <u>Organize and display data using appropriate methods</u> (including spreadsheets) to detect patterns and departures from patterns.</p> <p><b>L1.3.</b> <u>Compute and explain summary statistics</u> for distributions of data including measures of center (mean, median) and spread (range, percentiles, variance, standard deviation).</p> <p><b>L1.4.</b> Compare data sets using graphs and summary statistics.</p>
<p><b>St3. Interpret data displays and summaries critically; draw</b></p>	<p><b>L1.2.</b> <u>Read and interpret</u> tables, charts and graphs.</p>

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<p><b>conclusions and develop recommendations.</b>            Include paying attention to the context of the data, interpolating or extrapolating judiciously and examining the effects of extreme values of the data on summary statistics of center and spread. Include data sets that follow a normal distribution. Include observing and interpreting linear trends in bivariate quantitative data.</p>	<p><b>L2.1.</b> <u>Evaluate reports based on data</u> published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed.</p> <p><b>L3.1.</b> Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection and the <u>conclusions that can rightfully be made.</u></p> <p><b>L1.5.</b> Create scatter plots, analyze patterns and describe relationships in paired data.</p> <p><b>L1.6.</b> Know the characteristics of the Gaussian <u>normal distribution</u> (bell-shaped curve).</p>
<p><b>St4. Draw statistical conclusions involving population means or proportions using sample data.</b>            Conclusions should be based on simulations or other informal techniques, rather than formulas.</p>	
<p><b>St5. Evaluate reports based on data.</b>            Include looking for bias or flaws in way the data were gathered or presented, as well as unwarranted conclusions, such as claims that confuse correlation with causation.</p>	<p><b>L2.1.</b> Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed.</p> <p><b>L2.2.</b> Identify and explain misleading uses of data.</p> <p><b>L2.3.</b> Recognize when arguments based on data confuse correlation with causation.</p> <p><b>L3.1.</b> Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection and the conclusions that can rightfully be made.</p>



## American Diploma Project Benchmarks not covered in the Common Core College and Career Readiness Standards for Mathematics (Draft 09/21/09)

*(Note: 10 of these 12 ADP Benchmarks not covered in the Common Core College and Career Readiness Standards for Mathematics are denoted with asterisks, meaning that while recommended for all students they are required only for students intending to take calculus.)*

### J. Algebra

**J2.4.** \* Combine functions by composition, as well as by addition, subtraction, multiplication and division.

**J2.5.** \* Identify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line  $y = x$ .

**J2.6.** \* Know the inverse of an exponential function is a logarithm, prove basic properties of a logarithm using properties of its inverse and apply those properties to solve problems.

**J3.4.** \* Solve systems of three linear equations in three variables.

**J4.6.** \* Graph ellipses and hyperbolas whose axes are parallel to the  $x$  and  $y$  axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.

**J5.5.** \* Recognize and solve problems that can be modeled using an exponential function but whose solution requires facility with logarithms, such as exponential growth and decay problems.

**J6.** \* Understand the binomial theorem and its connections to combinatorics, Pascal's triangle and probability.

### K. Geometry

**K1.3.** Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true.

**K11.3.** Use the standard formula for the area of a triangle,  $A = \frac{1}{2}bh$ , to explain the area formula,  $A = \frac{1}{2}absinC$  where  $a$  and  $b$  are the lengths of two sides of a triangle and  $C$  is the measure of the included angle formed by these two sides, and use it to find the area of a triangle when given the lengths of two of its sides and the included angle.

**K12.1.** \* Know that the trigonometric functions sine and cosine, and thus all trigonometric functions, can be extended to periodic functions on the real line by defining them as functions on the unit circle, that radian measure of an angle between 0 and 360 degrees is the arc length of the unit circle subtended by that central angle, and that by similarity, the arc length  $s$  of a circle of radius  $r$  subtended by a central angle of measure  $t$  radians is  $s = rt$ .

**K12.2.** \* Know and use the basic identities, such as  $\sin^2(x) + \cos^2(x) = 1$  and  $\cos(\pi/2 - x) = \sin(x)$  and formulas for sine and cosine, such as addition and double angle formulas.

**K12.4.** \* Know and use the law of cosines and the law of sines to find missing sides and angles of a triangle.