**Lesson/Unit Name:** Congruence, Constructions and Proof  
**Content Area:** Mathematics  
**Grade Level:** Integrated Math I

### Dimension I – Alignment to the Depth of the CCSS

<table>
<thead>
<tr>
<th>The lesson/unit aligns with the letter and spirit of the CCSS:</th>
<th>Targets Standards:</th>
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<tbody>
<tr>
<td>✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.</td>
<td>The CCSS mathematics standards are clearly identified in each task. Core standards are identified, and when appropriate, related standards are also listed. The lessons are designed to insure that students reach a deep level of understanding of the selected standards (G.CO.1, G.CO.2, G.CO.3, G.CO.4, G.CO.5, G.CO.6, G.CO.7, G.CO.8, G.GPE.3). Development of the concepts is in line with the intent of the CCSS Mathematics content standards in that understanding of congruence is built from solid understanding of rigid transformations. Strong connections to proof of these concepts are central to the lessons through thoughtful and continual justifications. Lessons also seem to follow the CCSS standards to the precise letter of the written standard. For example, G.CO.4 states that students must &quot;develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.&quot; This is exactly what lesson 6.4 asks students to do in the activities.</td>
</tr>
<tr>
<td>☐ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.</td>
<td>Standards for Mathematical Practice:</td>
</tr>
<tr>
<td>✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.</td>
<td>While not explicitly identified, SMP are embedded within each lesson and teacher notes. For example, page 6 of teacher notes specifically draws attention to the need for students to recognize the need for precise definitions. This reflects the purposeful planning of student behaviors that require attention to precision. Another example occurs on page 7 where the teacher notes say, &quot;provide multiple tools for students to do this work, such as transparencies or tracing paper, protractors...&quot; and on page 36 with, &quot;What patterns do you notice when...&quot; e examples of using tools strategically and looking for repeated reasoning respectively. An upgrade that authors should consider would be to explicitly identify these opportunities so that teachers can easily locate these practices and purposefully plan for them.</td>
</tr>
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</table>

### Balance:
Materials include balanced activities that encourage conceptual understanding as well as ample opportunities for practice to build procedural skill. Primary lessons focus heavily, though appropriately, on conceptual development of key concepts aiming for deep understanding. The "practice" lessons allow opportunities to ensure fluency and to practice the understandings that have been developed. These lessons also refine student understanding by providing unique tasks or special aspects of the concept to ensure deep understanding. The other aspect of this unit that balances the conceptual development are the "Ready, Set, Go" resources as they are built to maintain prior skills and to build fluency in current concepts.

Rating: **3 – Meets most to all of the criteria in the dimension**
### Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.

- **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.

- **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
  - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
  - **Conceptual Understanding:** Develops students’ conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
  - **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

#### Focus:

Each set of standards, those related to congruence through transformations and those related to construction, targeted in this module are major work of the grade and are explored and refined through a research-based learning cycle where students "develop, solidify, and practice" their understanding of the key concept. Appropriately high expectations are set beginning with the first lesson, an assessment of prior knowledge, as it asks students to reflect over line $y=1/2x+16$, a non-traditional choice for a line of reflection on the coordinate plane.

#### Coherence:

Lessons throughout the unit connect to prior learning (parallel lines have same slope p.9) and to future learning. For example, page 11 of the student pages indicate to students that they will extend their justifications to formal proof in next module/unit. However, this indication that formal proof is explored more in depth at a later time does not limit the expectation that students begin to use formal definitions to justify "why" these transformations work as they do, laying groundwork for future learning. The unit as a whole connects the major work and supporting work in a balanced, appropriate manner. For example, task two connects major standard G.GPE.5 with supporting standard G.CO.1.

#### Rigor:

Much of the module is based on a common theme, leading to many of the tasks being application based. Students are often provided situations in which they are making conjectures about the mathematics as they explore and develop conceptual understanding and they are frequently submerged in an application in which they must apply new knowledge and skills to solidify and refine their understanding.

**Rating:** 3 – Meets most to all of the criteria in the dimension

### Dimension III – Instructional Supports

The lesson/unit is responsive to varied student learning needs:

- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.

- Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.

**Clear and Sufficient Guidance**

This unit of instruction is designed around a research-based learning cycle where students Develop, Solidify, and Practice Understanding. Additionally, each lesson follows a research-based teaching cycle where each lesson contains a launch, an exploration, and a discussion. The teacher notes clearly identify each aspect of these teaching and learning cycles and provide detailed descriptions, without being scripted, so that teachers can successfully implement the lesson.

Each activity provides in-depth description of the activity, student discourse, and key concepts as related to prior and future learning. Possible student
- Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
- Addresses instructional expectations and is easy to understand and use.
- Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
  - Supports diverse cultural and linguistic backgrounds, interests and styles.
  - Provides extra supports for students working below grade level.
  - Provides extensions for students with high interest or working above grade level.

A unit or longer lesson should:
- Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
- Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Lessons also provide options for teachers when facilitating discussions, "If either idea is missing, ask additional questions to prompt for it. For example, "How did you know how far away from the center point (or the reflecting line) this image point should be?"

Precision, Productive Struggle, and Expectations
Students are continually engaged in productive struggle in each and every lesson, and this is described in the teacher notes with phrases such as, "press upon students to...." Instructional expectations are clearly addressed ("Watch for two different strategies to emerge..." page 8). As described in earlier dimensions, attention to precision is central to all lessons, encouraging students to use precise and accurate mathematics and academic language.

Differentiation
While this unit allows for teachers to modify instruction and meet needs of many types of learners, it does not explicitly address needs of students from diverse cultural or academic backgrounds. Scaffolding, differentiation, intervention and/or support needs to be more evident in the tasks. Adding in teacher and material support for these students will strengthen teacher tools and student understanding throughout the unit.

Longer units
A variety of instructional approaches are used throughout the unit including the use of several different types of models and tools, error analysis, and a wide-range of questioning and tasks. The develop, solidify, practice learning cycle naturally removes supports through the cycle and provides an effective sequence of learning that deepens over time. The “Ready-Set-Go” materials provide support for maintaining and building fluency in procedural skills.

Rating: 3 – Meets most to all of the criteria in the dimension

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:
- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:
- Use varied modes of curriculum-embedded assessments that may include pre-, formative, methods, including potential misconceptions, are identified, as are teacher strategies to guide student work and explicit definitions or descriptions of key concepts (p. 6). Specific suggestions are provided for debriefing student findings (teacher page 9), and frequent references to possible technologies are made. However, additional support on the incorporation of technology would be helpful.

The assessments throughout the unit are found in many formative ways throughout the lessons, and evidence of student understanding can be gathered using these materials. This unit explicitly draws attention to use of formative assessment to define student prior knowledge by modeling an entire lesson to be used as a formative assessment (lesson 1). Specific suggestions are given to the teacher throughout each and every lesson about what to look for during student performance, combining performance assessment with teacher observation (a personal communication assessment). An end-of-module assessment is also provided. These demonstrate that varied forms of assessment are central to the unit. However, many of the assessments embedded in lessons are not explicitly identified as such. Writers should consider making assessment aspects visible to teachers to support continual and purposeful formative assessment. Additionally, rubrics, answer keys, and scoring guidelines are not provided.
summative and self-assessment measures.

Rating: 2 – Meets many of the criteria in the dimension

Summary Comments

Overall, this unit is fantastic. It encourages student exploration and discussion and provides an excellent model of common-core aligned instruction.

Future revisions need to include specific mentions of the Standards for Mathematical Practice, specific supports for students needing additional support, and specific attention to assessment.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:
3: Meets most to all of the criteria in the dimension
2: Meets many of the criteria in the dimension
1: Meets some of the criteria in the dimension
0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:
E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (total 11 – 12)
E/I: Exemplar if Improved – Aligned and needs some improvement in one or more dimensions (total 8 – 10)
R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (total 3 – 7)
N: Not Ready to Review – Not aligned and does not meet criteria (total 0 – 2)

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:
3: Exemplifies CCSS Quality - meets the standard described by criteria in the dimension, as explained in criterion-based observations.
2: Approaching CCSS Quality - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
1: Developing toward CCSS Quality - needs significant revision, as suggested in criterion-based observations.
0: Not representing CCSS Quality - does not address the criteria in the dimension.

Descriptor for Overall Ratings:
E: Exemplifies CCSS Quality – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
E/I: Approaching CCSS Quality – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
R: Developing toward CCSS Quality – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.
N: Not representing CCSS Quality – Not aligned and does not address criteria.