Lesson/Unit Name: Expressions and Equations  
Content Area: Mathematics  
Grade Level: 7

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
- Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

This module, which consists of 26 lessons, targets CCSS mathematics content standards 7.EE.A1 & 2; 7.EE.B.3 & 7; 7.EE.B.4, and 7.G.B.4,5 & 6. The module is broken into three Topics. Topic A focuses on using properties of operations to generate equivalent expressions, Topic B targets the solving of problems using expressions, equations, and inequalities, then Topic C involves the use of equations and inequalities to solve geometry problems. If implemented with fidelity as written, the depth of the majority of targeted standards will be met. Foundational prerequisite CCSS mathematics standards are listed on pp. 6-8 as information for teachers who may be working with students lacking the knowledge, understandings, and skills that should come prior to this module.

The Standards for Mathematical Practice (MPs) that are central to the teaching and learning in the module are listed in the module Overview on pp. 8-9. These are MP2, MP4, MP6, MP7, and MP8. The description for each MP is specific to the content addressed in the lessons and is handled in a grade-appropriate way. The reviewers of the module felt that both students and teachers might gain greater insight and conceptual understanding if Standard for Mathematical Practice 1 - "Make sense of problems and persevere in solving them", and Standard for Mathematical Practice 3 - "Construct viable arguments and critique the reasoning of others" were more integral to the module instructional strategies. It appears that there is more emphasis on MP8, repeated reasoning, throughout the module with almost no mention of MPs 1, 3, and 5 - making sense of problems and productive struggle, student reasoning with each other, and the use of appropriate tools strategically. More will be said about this in Dimension III of this review.

There is a balance of mathematical procedures and deeper conceptual understanding throughout the module. In Topic B, instructional strategies continue to use bar diagrams and other types of pictorial representations from earlier grades for students to conceptually understand the meaning of problems and relate them to the more abstract numerical expressions and equations for these same problems. Students are gradually guided in their discovery that some problems would more efficiently be solved algebraically, thus developing value for the art of setting up and solving real-world problems using algebraic equations.

Rating: 3 – Meets most to all of the criteria in the dimension

Rating: E – Exemplar

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

FOCUS: Two-thirds of this module focus on clusters A and B in the Grade 7 Expressions and Equations domain, which are major work of Grade 7. There is a particularly in-depth treatment of 7.EE.4a "Compare an algebraic
Focus: Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.

Coherence: The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.

Rigor: Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

- **Application**: Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
- **Conceptual Understanding**: Develops students’ conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
- **Procedural Skill and Fluency**: Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Solution to an arithmetic solution, identifying the sequence of the operation used in each approach" in lessons 7-9. Also, the emphasis in later lessons on writing equivalent equations or equivalent inequalities by clearing the fraction or decimal helps develop 7.EE.2 and will be useful to students in later mathematics. However, reviewers felt that some of the problems included in Topic C seem to be rather advanced and more in line with G.GMD.3 than 7th grade work. In particular, these are the problems involving containers such as Example 3 in Lesson 22.

COHERENCE: As stated previously for Dimension I of this review, the Overview of the module lists connected foundational standards from previous grades. The module builds heavily on 6.EE and 6.G standards and teacher notes within the lessons identify how to build on this knowledge. Within-grade coherence is evident in the opening lessons as students review that the same properties of operations for rational numbers hold for operations with variables. Knowledge of rational number operations from Module 2 is demonstrated as students collect like terms containing both positive and negative constants and coefficients. Adding the 7.G.4 - 7.G.6 standards as part of this module ties the work with solving for measurements using geometric formulas in tightly with the work on equations, rather than separating the geometry work out in a different unit. This is a great example of cross-domain coherence and the connections that exist in mathematics understandings and skills across algebra and geometry.

RIGOR: With regards to the shift of rigor, there is evidence of application as the majority of problems in Topics B and C are real-world problems presented in context. These problems ask students to find solutions using their knowledge of expressions and equations. The contexts include consecutive numbers, total cost, age comparisons, distance/rate/time, perimeter, area, surface area, and volume and missing angle measurements to name just a few.

Topics B and C have many instructional activities that encourage conceptual understanding through the use of pictorial representations and bar diagrams. The author might reflect on the fact that Topic A is mainly procedural. Conceptual understanding of the properties of operations and "why" these properties also work for variables could be enhanced with more intentional modeling using the suggested tools and representations listed on p. 11 of the module Overview. These include area models and number lines in addition to the bar diagrams already used in the module. The author should also consider the use of algebra tiles and pan balance activities to emphasize the conceptual meaning of the properties of operations with variables and the steps in solving equations.

Procedural skill and fluency are addressed throughout the module. The emphasis on "if-then" moves in solving equations and inequalities ties raises procedural skill to more critical thinking. There are several places where "sprint exercises" are included in the lessons to encourage fluency. However, the reviewers would like the author to consider if the time constraints put on these sprint exercises are realistic and if they reflect the meaning of fluency as defined by the standards. Do they help or discourage students? The author might consider activities like number talks around the procedural skills to develop student flexibility in thinking and more student discourse about the multiple solution paths that are possible to arrive at an accurate solution when solving problems.
The lesson/unit is responsive to varied student learning needs:

- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
- Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
- Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
- Addresses instructional expectations and is easy to understand and use.
- Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
  - Supports diverse cultural and linguistic backgrounds, interests and styles.
  - Provides extra supports for students working below grade level.
  - Provides extensions for students with high interest or working above grade level.

A unit or longer lesson should:

- Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
- Gradually removes supports, requiring students to demonstrate their mathematical understanding independently.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

The reviewers found that the module provided adequate support for teaching and learning. Within each lesson there are defined student outcomes, lesson notes, class work, exercises, closure and an exit ticket. Overall the format is easy to follow and the module Overview provides a strong sense of the unit before beginning to look at the specifics of the lessons. The teacher notes include questions to ask during the lesson and are appropriate as the lesson progresses.

The lesson problems are challenging and encourage the precise and accurate use of both mathematics and academic language. One exception was found in the Overview with a reference to "reducing" within a problem. On p. 14, this was clarified as "An expression in standard form is the equivalent of what is traditionally referred to as a "simplified" expression." After that reference, this module does not utilize the term "simplify" when writing equivalent expressions, but rather asks students to "put an expression in standard form" or "expand the expression and combine like terms." Students must know that the term "simplify" will be seen outside of this curriculum and that the term is directing them to write an expression in standard form.

Opportunities for productive struggle are limited because most of the lessons are teacher-directed. The author might consider guiding the teacher to use more of the problem set examples to intentionally engage students in productive struggle and conversation as is expected with MP1 and MP3. Providing more opportunities for students to engage in open-ended inquiry will allow for productive struggle and provide opportunities to develop more reasoning and critiquing skills. See p. 56 as an example of an exit ticket that might have been used as an "entry" activity to drive student thinking, encourage productive struggle, emphasize perseverance and the construction of viable arguments and critiquing of the reasoning of others, thus requiring the more of the mathematical practices. The author might reflect on if there are too many tasks and not enough time for productive struggle with these tasks.

Just one example of a teacher-directed lesson that should be studied more deeply by the author is Lesson 16 - The Most Famous Ratio of All. With one demonstration of the tire print of the rotation of a bicycle wheel, students are to believe that the ratio of the circumference to the diameter is always pi. The author might consider a class activity that collects and analyzes the relationship between the circumference and diameter of many circular objects for students to discover pi. There could be a missed opportunity here for students to revisit some of their SP standards from Grade 6, especially for Grade 6 SP Cluster B - Summarize and describe data distributions.

It is recommended by the reviewers that the author provide more teaching and learning supports for students working below grade level and students from varying cultural and linguistic backgrounds. There are a few attempts at scaffolding in small notes, such as that on p. 29, but these are not adequate for the diverse needs of students. Although connections to prior grade level standards are explicitly stated in the module Overview, the reviewers recommend more strategies for how teachers might activate this prior knowledge to support the Grade 7 learning targets for this module. The authors might also consider more notes and strategic questions that teachers might use to surface common student misconceptions and errors and strategies to address them throughout the lesson. Intentional
questioning and "listening" to student discussion surrounding processes for solving problems and equations are essential teacher practices to address and present those questions that enable students to question their own thinking. It is also recommended that the author review the expectations of problem sets and exit tickets given the time frame of 45 allotted minutes for lessons. Also consider a review of the exit tickets, so that expectations are realistic for students given a short time frame. For example, on p. 153 students are asked to describe, in just one sentence, all the relevant angle relationships in the accompanying complex diagram.

Lastly, this longer module does present a sequence of learning where concepts or skills advance and deepen over time. The module also expects and provides guidelines for procedural skill and fluency with core calculations and procedures, but the authors might consider more conceptual understanding opportunities for those students who are kinesthetic and visual learners. Most research shows that students progress from concrete, to representational/pictorial, to abstract in their learning of mathematics concepts, and the expressions and equations concepts in this module are no exception. The reviewers highly recommend a greater mix of instructional approaches that include using more hands-on and visual representations (algebra tiles, number lines, area models, etc.), more strategies for accountable student discourse, such as effective cooperative learning protocols, and more opportunities to recognize and celebrate multiple solution paths.

Rating: 2 – Meets many of the criteria in the dimension

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:
✓ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:
✓ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

The module regularly assesses whether students are mastering standards-based content and skills. There are several levels of formative assessment embedded in the lessons including teacher questioning, class problems, and exit tickets. The author might consider a few more directions to teachers as to how to interpret the results of exit tickets and class problems misconceptions and so that they can inform future instruction and make mid-course instructional adjustments when students are not progressing as anticipated. Mid-Module and End-of-Module summative assessments are also provided in the module.

A strong asset in this module are the answer keys and rubrics that help teachers understand the expectations of the module and score student work in a consistent, constructive, and formative way.

The author might consider a pre-assessment prior to each topic to gain information about student misconceptions and what students might already know prior to teaching. This information would help guide teachers to make adjustments to lessons as dictated by their class needs. A post-topic short formative assessment would also help to gauge student growth and again, make the needed interventions in the upcoming lessons. Student strategies to assess their own progress with regards to the learning targets in the module are also appropriate for Grade 7 students so that they take more ownership of their learning and growth for the mathematics content and skills.

Rating: 3 – Meets most to all of the criteria in the dimension

Summary Comments
Overall, this is a module that will be effective in the teaching and learning of the targeted content and practice standards. The sequence of learning is conducive to student understanding and the module organization is easy for teachers to follow. Content and expectations meet the depth of the focus Expressions and Equations standards, which are major work for Grade 7. Previous grade, within grade, and cross-domain coherence is evident throughout the module. Rigor is evident in the expectations for procedural skill and fluency and there are many opportunities for applying the understandings of the content standards in real-world problem contexts.

The reviewers suggest that the authors consider:
- the amount of content in each lesson and more guidance on if all of this content can be realistically taught and learned in a 45-minute class period. The authors might consider which lesson activities are non-negotiable and which are "optional if time allows". An examination of the procedural skill "sprints" is also encouraged as to how both teachers and students might use these activities to measure and reward growth rather than total mastery.
- more intentional lesson opportunities for student discourse and more learning activities for students to work effectively together to solve problems. Most of the lessons are very teacher directed and do not capitalize on or provide time for student inquiry of concepts and productive struggle to discover the mathematics behind the "rules" for simplifying expressions and solving equations/geometric formulas for the unknown.
- the use of more hands-on and visual conceptual tools and representations, such as algebra tiles, number lines, and online pan balance applets, etc. for the conceptual understanding of the properties of operations and the steps for solving equations.
- more guidance for teachers on how to identify and address student error patterns and misconceptions that arise during instruction and in other assessments, such as exit tickets.
- a module strategy for students to identify their learning goals/targets for the module and assess their own progress throughout the module.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:
3: Meets most to all of the criteria in the dimension
2: Meets many of the criteria in the dimension
1: Meets some of the criteria in the dimension
0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:
E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (total 11 – 12)
E/I: Exemplar if Improved – Aligned and needs some improvement in one or more dimensions (total 8 – 10)
R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (total 3 – 7)
N: Not Ready to Review – Not aligned and does not meet criteria (total 0 – 2)

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:
3: Exemplifies CCSS Quality - meets the standard described by criteria in the dimension, as explained in criterion-based observations.
2: Approaching CCSS Quality - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
1: Developing toward CCSS Quality - needs significant revision, as suggested in criterion-based observations.
0: Not representing CCSS Quality - does not address the criteria in the dimension.

Descriptor for Overall Ratings:
E: Exemplifies CCSS Quality – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
E/I: Approaching CCSS Quality – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
R: Developing toward CCSS Quality – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.
N: Not representing CCSS Quality – Not aligned and does not address criteria.