Lesson/Unit Name: Rates, Ratios and Proportions
Content Area: Mathematics
Grade Level: 6

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
- Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

This is a unit designed for grade 6 to provide instruction about ratios, rates, and proportions across 6-7 weeks. The unit focuses on content standards 6.RP.1, 6.RP.2, and 6.RP.3 under "Understand ratio concepts and use ratio reasoning to solve problems." as well as 6.EE.9 under "Represent and analyze quantitative relationships between dependent and independent variables." These standards are considered major content in grade 6. The lessons are designed to target these standards and help students develop a depth of understanding about ratios, rates, and proportions by making connections between these standards. In the lessons contained in this review, 6.RP.2 is not addressed to the full depth of the standard. The lesson could be improved, however, with more attention paid to equivalent ratios, and activities that motivate the need for unit rate.

The unit incorporates all eight of the Standards for Mathematical Practice with emphasis concentrated on SMP #1 - "Make sense of problems and persevere in solving them.", SMP #4 - "Model with mathematics.", SMP #6 - "Attend to precision.", SMP #7 - "Look for and make use of structure.", and SMP #8 - "Look for and express regularity in repeated reasoning."

Additional detail about what the Standards for Mathematical Practice will look like in the classroom are provided in each overview as "Math Practice Look-Fors".

This unit provides a balance of deeper conceptual understanding with mathematical procedures. Students spend time using concrete objects and visual representations to develop conceptual understanding of ratios and ratio work. There are ample opportunities for students to develop procedural skill in the various activities throughout the model lesson.

Rating: 3 – Meets most to all of the criteria in the dimension

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- Focus: Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- Coherence: The content develops through reasoning about the new concepts on the basis of prior knowledge.

Focus
"Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems" (CCSS-Math, Grade 6, Introduction) is one of four important ideas emphasized in grade 6 mathematics. This unit focuses on three of the four 6.RP standards which are major content in grade 6. The concept of ratio and models for working with ratios are developed over several days of segments in the model lesson. For instance, in the Exploring Unit Rate activity the unit rates are all equal. The focus of this unit could be improved by including some activities about equivalent ratios and also some activities where students compare ratios.

Coherence
of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.

✓ Rigor: Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

- **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.

- **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.

- **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

The overview provides clear connections to previous learning, such as multiples, factors, and divisibility rules learned in elementary school. The Pre-assessment addresses the manner in which the educator can determine if the students have the pre-requisite skills and understanding to pursue the content within the unit.

The unit prepares students with a foundational understanding of ratio and rate that will be used in future courses, such as algebra and geometry.

Rigor is addressed in the lesson description and is evidenced in the lesson notes and activities.

Conceptual

Students get to use physical models and visual representations to explore ratios and unit rates without jumping quickly to an algorithm, which helps develop conceptual understanding. Students are developing the concepts of ratio and rate by attending to the language of the real world situations and representing the situations with diagrams and algebraic symbols. (6.RP.1, 6.RP.2)

Procedural

While procedural skill and fluency are not a focus for these standards, students get opportunities to work with ratios in several lesson segments, thus developing procedural skill at working with ratios. Students will use the procedures of translating situations into diagrams and/or algebraic expressions or equations. (6.RP.1, 6.RP.2)

Application

Essentially all of the problems students encounter in this unit are based on context and application. Engaging tasks, such as building a life-size Origami Yoda, provide a context for real-world application. Ratio and Proportions will be used in the context of real world problems to build mathematical models. (6.RP 3)

Rating: 3 – Meets most to all of the criteria in the dimension

**Dimension III – Instructional Supports**

The lesson/unit is responsive to varied student learning needs:

- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
- Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
- Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
- Addresses instructional expectations and is easy to understand and use.

Clear and Sufficient Guidance

The lesson overview provides critical information to inform educators regarding standards, mathematical practices, the specific learning targets in student-friendly language, and a step-by-step guide to utilize the lesson, including questions to ask students, and grouping strategies. During the inquiry segments, it would be helpful to guide educators in the ultimate goal as to what should be accomplished through the productive struggle. When sending students to explore the websites, it might be beneficial to have a purpose for the exploration. There seems to be some disconnect to the activities that follow the exploration.

Precise and accurate mathematics

The unit uses and expects the use of precise and accurate mathematics. Important vocabulary for each lesson is highlighted in the lesson overview. It might be helpful to include definitions for each of these terms.

Engage in productive struggle

Students engage in a variety of problem solving tasks and lessons provide some scaffolding to allow students to engage in some productive struggle. Each lesson denotes the areas in the lesson that address rigor through
- Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
  - Supports diverse cultural and linguistic backgrounds, interests and styles.
  - Provides extra supports for students working below grade level.
  - Provides extensions for students with high interest or working above grade level.

**A unit or longer lesson should:**

- Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
- Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

conceptual understanding, procedural knowledge, and opportunities for application. Throughout the unit, the notes to teachers help bridge the productive struggle to the learning of new mathematical ideas. "Potential Pitfalls" is a useful way to help teachers anticipate potential student misconceptions. It would help the lesson if there were more guidance to the teachers about questioning students during the activities to foster productive struggle and elicit mathematical thinking.

**Ease of Use**

The LiveBinder structure requires some learning of the way to maneuver within it, but the parts are easily obtainable once you know where to look. It also may help to provide teachers instructions for how to use the checklist observations, as novice teachers may not be familiar with how to use such a tool. There are also some clarification issues that need to be addressed:

1) In Segment 8, the needed resource is listed as "Lesson 1 Problem Solving with Ratios and Rates". There doesn't appear to be a resource by that name in the LiveBinder.

2) A 'skeleton' is referred to in Segment 8, but there is no clarification about what that is or where to find it. Consider clarifying this.

3) It is unclear when the Pre/Post Unit assessment should be given, as it doesn't appear on the "Assessment Plan" or in the lesson plan itself. Consider providing more guidance around this.

4) On the pre-assessment radar, the word proportion appears twice. Consider editing that. Also, proportions aren't introduced until 7th grade, so it may not make sense to include this term on the radar.

5) In segment 2, it is not clear how the vocabulary term "ratio" is introduced. If it occurs during the "Matching Ratios" activity, it might be useful to include guiding questions so teachers have a sense of how to engage students in a class discussion about the vocabulary. It would also be helpful to include examples of what the teacher might expect students to say during this conversation to know that the conversation is headed in the right direction.

**Scaffolding and differentiation**

There is some evidence for providing differentiation and support for a broad range of learners. For example, there is a note in Segment 7 that the students could determine their own texting rate as an enrichment task. The suggestions for remediation include using manipulatives, such as Cuisenaire rods. The suggestions for helping English Language Learners typically include rephrasing or working with a buddy. Consider adding more specific vocabulary and ELL strategies for each segment.

**Effective sequencing**

The lesson flow feels a bit disjointed, like a series of activities, without a cohesive thread. Guidance to teachers about how to connect these activities through discussion and/or reordering some of the activities seems advisable. In Segment 2, students are first solving ratio problems using unifix cubes and tape diagrams (which they may or may not have been introduced to) and then they are doing an activity to match the different ways of writing ratios. These activities should be in the opposite order. Segment 3 also feels disjointed, as the first step has to do with the Thinking Blocks activity, and the second part has to do with ice cream ratios where students are plotting points for a ratio relationship on a coordinate grid. The Recess Choice activity should be done at the beginning of the unit, since all students are doing is writing ratios based on values in the graph. This would be a good activity to have students practice writing ratios in different ways. The one-inch tall activity, while engaging,
shouldn’t take much class time. Perhaps consider using it as an opening activity to hook students into wanting to learn about ratios.

Rating: 2 – Meets many of the criteria in the dimension

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:

- Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

Student Proficiency
The pre/post assessments provide evidence of student progress towards the identified learning targets.

Bias
The assessments appear to be unbiased with context that is appropriate for the grade and diverse cultures.

Scoring Guidelines
Not all of the assessments and activities have answer keys or rubrics. Those that do have answer keys may not provide enough guidance for teachers to interpret student performance. This lesson would be improved by including answer keys for all activities and assessments, and providing guidance as to how to interpret the students' performance. The Multiple Representation Observation Checklist is good, but there is little direction as to how it is to be utilized.

Varied assessments
The lesson includes a wide variety of assessments including the two parts to the pre-assessment, the ongoing formative assessments, observation checklists, summative assessments and the self-assessment. The overall assessment plan for the unit is particularly helpful in seeing how and when student mastery will be assessed.

Rating: 2 – Meets many of the criteria in the dimension

Summary Comments

This lesson clearly targets many of the 6.RP standards, as well as 6.EE.9. There are a variety of engaging activities to use in class, as well as a unit assessment plan to guide teachers. The lessons develop conceptual understanding through the use of models such as double number lines and strip diagrams.

This unit could be improved by providing more guidance to teachers about how to develop the initial definitions of ratio so that students clearly understand what a ratio is. Also, the sequencing of activities could be adjusted to make for a better flow and less disjointed lessons. Improved suggestions to teachers about how to support English language learners are needed, particularly since every problem in the unit is based in a context. Finally, the unit could be improved by including answer keys and other supports to teachers to interpret student performance.

This unit has many promising aspects. With some clarification in Dimension III (Instructional Supports) and Dimension IV (Assessment), this lesson could become an Exemplar.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:
3: Meets most to all of the criteria in the dimension
2: Meets many of the criteria in the dimension
1: Meets some of the criteria in the dimension
0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:
E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (total 11 – 12)
E/I: Exemplar if Improved – Aligned and needs some improvement in one or more dimensions (total 8 – 10)
R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (total 3 – 7)
N: Not Ready to Review – Not aligned and does not meet criteria (total 0 – 2)

Rating Descriptors
Descriptors for Dimensions I, II, III, IV:
3: Exemplifies CCSS Quality - meets the standard described by criteria in the dimension, as explained in criterion-based observations.
2: Approaching CCSS Quality - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
1: Developing toward CCSS Quality - needs significant revision, as suggested in criterion-based observations.
0: Not representing CCSS Quality - does not address the criteria in the dimension.

Descriptor for Overall Ratings:
E: Exemplifies CCSS Quality – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
E/I: Approaching CCSS Quality – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
R: Developing toward CCSS Quality – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.
N: Not representing CCSS Quality – Not aligned and does not address criteria.