EQuIP Review Feedback

Lesson/Unit Name: Toothpick Patterns
Content Area: Mathematics
Grade Level: Grade 6

Overall Rating:

E/I

Exemplar if Improved

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS: ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ❑ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	This lesson is designed to incorporate the following cluster of CCSS standards in the Domain of Expressions and Equations: 6.EE.A.2a, b, and c: Write, read and evaluate expressions in which letters stand for numbers. 6. EE.A.3: Apply the properties of operations to generate equivalent expressions. 6. EE.A.4: Identify when two expressions are equivalent. 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. While these standards are connected to this lesson, the reviewers find the lesson does not address all of the identified standards to the expected depth. For example, the lesson fully addresses 6.EE.A.2a in that it has students writing expressions using letters in the place of unknown quantities to represent the number of toothpicks in each design stage (and later the number of stones). The reviewers did not see evidence of standard 6.EE.A.2b; 'identify parts of an expressions using mathematical terms'. Students are evaluating expressions, 6.EE.A.2c for various values of "n", however, none of the expressions in the lesson contained exponents. The reviewers understand that this is a lesson about linear expressions, but feel that somewhere it should be noted that the standard is only partially addressed. The reviewers find 6.EE.A.3 and 6.EE.A.4 are fully addressed as various student generated expressions are presented and identified as being equivalent in the discussion phase. The reviewers discussed standard 6.EE.C.9. This lesson suggests the use of variables to create a formula. There is no expectation in this lesson for students neither to create the variables in this lesson nor to analyze the quantitative relationship between the variables. Students are prompted in handout #1 (phase 2) to create a table and describe a pattern. In phase 3 students are asked, "How many (toothpicks) in the nth design?" An expression would suffice to describe the number of toothpicks but students are asked to create formulas prompted by T =, to explain the toothpick patterns. This lesson has no expectation for defining the variables in terms of dependent and independent nor analyzing the relationship between the independent and dependent variables using graphs or tables (only Poster A created a table). The reviewers do not see standard 6.EE.A.9 developed in this lesson however they do see standard 6.EE.B.6 (Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set). This standard is addressed in the lesson

several times. In the workshop phase and in the final focus problem phase, the lesson expects students to use variables to represent a number quantity and write a variable expression to describe the total number of toothpicks.

The CCSS not only define a set of grade level content expectations but also define eight K-12 Standards for Mathematical Practice (SfMP) that describe the ways students ought to be engaged with the mathematics standards. The reviewers noted the lesson engages students in several ways with the content standards in the lesson, however, no SfMP are identified. The reviewers noted for example that as students developed their expressions for the 'n'th design they were engaged in the content through SfMP.8, Look for and express regularity in repeated reasoning,

The lesson presents a balance of conceptual understanding and procedural fluency. It begins with an interesting problem that students can model with physical objects. Students are asked to understand the concept in the real life scenarios and build upon that to look for the mathematics in the problem. From there, the lesson expands to connect how this situation might be represented with mathematical symbols and students must justify their thinking. Students create charts and formulas to model the scenarios and must understand the concepts of creating organized charts and looking for patterns in the resulting numbers of toothpicks and stones.

Rating: **2 – Meets many of the criteria in the dimension**

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:
 	
✓ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
  – **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
  – **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions,
	This lesson shows evidence of the three key shifts that are indicated in the CCSS.
	
	A focus in grade 6 mathematics and reflected in this lesson, is developing an understanding of variables and their use in writing mathematical expressions, formulas, and equations to solve problems as well as developing an understanding of equivalent expressions by rewriting expressions using properties of operations. This lesson makes a clear connection in both the toothpick design problem and the garden size problem with creating variable expressions that describe each problem.
	
	This lesson provides connections to previous learning, such as using properties of operations. The lesson also coherently develops student understanding of variable expressions and equivalent expressions through each phase of the lesson as first students are asked to write expressions for the number of toothpicks at each design stage and then to identify the equivalence of expressions using the properties of operations to rewrite expressions as well as by substitution justify their equivalence. It is not stated in the lesson what previous knowledge students require, nor does it state what the follow up will be.
	
	There is a balance of application, conceptual understanding and procedural skill in the lesson. The expectation is that students will be able to solve a problem in a real world context by creating models, represented pictorially, by table, and mathematically. Students have the opportunity to engage in productive struggle while they work to find patterns. Questions such as, "Do some formulas show what is going on better than others?", blends conceptual understanding with procedural skill as students work to discover the patterns. Students must then work to represent their solution in a way that can be communicated to others.

- multiple representations and opportunities for students to write and speak about their understanding. – **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	

Rating: **3 – Meets most to all of the criteria in the dimension**

Dimension III – Instructional Supports

	There is clear step by step guidance for the teacher including links to tutorials in the "Teacher Tune Up" section. The lesson is chunked into 6 distinct instructional parts. The "Teacher Led Discussion" stresses pushing students thinking toward at higher levels, however, no guidance is given as to how that would be accomplished for students who are struggling. There are photos of student work samples to help teachers anticipate potential student responses. Additionally, there are handouts provided for the students' work.

The lesson/unit is responsive to varied student learning needs:
 	
✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
✓ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
✓ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
✓ Addresses instructional expectations and is easy to understand and use.
❒ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
   – Supports diverse cultural and linguistic backgrounds, interests and styles.
   – Provides extra supports for students working below grade level.
   – Provides extensions for students with high interest or working above grade level.
 	
A unit or longer lesson should:
❒ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
❒ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
❒ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
❒ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures
	The lesson provides opportunities for students to engage in productive struggle as well as opportunities to engage in the content through several Standards for Mathematical Practice. Instructional grouping strategies include pair work, small group work, and whole class discussions.
	
	The reviewers saw no evidence of:
	--Supports for diverse cultural and linguistic backgrounds, interests and styles.
	– Supports for students working below grade level.
	– Extensions for students with high interest or working above grade level

(when called for in the standards for the grade) to be performed quickly and accurately.	

Rating: **2 – Meets many of the criteria in the dimension**

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills: ❒ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. *A unit or longer lesson should:* ❒ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.	Assessment information from this 2 day lesson is primarily in the form of teacher observation of the student representations and the class discussion. The stated learning objectives on page 1 will help teachers focus their observations to informally know which students have mastered the content by the end of the lesson. The levels of current thinking on page 6 will provide guidelines for informally assessing student group thinking about the problem. The phase 6 problems allow students the opportunity to apply their understanding about writing expressions to a new situation with another student. However, the lesson lacks any opportunities for students to independently exhibit individual mathematical understanding of the content standards addressed in this lesson. The reviewers feel this to be an omission that impacts teacher's monitoring student progress in a critical area of grade 6 content.

Rating: **1 – Meets some of the criteria in the dimension**

Summary Comments

A particular strength of this lesson is in its alignment to the shifts in mathematics that the CCSS imply. Students have opportunities to engage with critical grade level content while collaborating with others to represent and discuss their mathematical thinking about a problem. Additionally, the presentation and chunking of the lesson into instructional components is commendable.

The reviewers suggest providing additional notes for teachers about prior knowledge students may need before beginning the lesson and connections in the lesson to that prior knowledge for students. (e.g. relating arithmetic expressions and algebraic expressions).

Students are provided with the opportunity to engage in productive struggle and work with the problem before having a class discussion about the problem. This lesson would be strengthened if the SfMP were identified for teachers and addressed in a more explicit manner. We strongly suggest at least adding SfMP.8 Look for and express regularity in repeated reasoning.

The reviewers recommend for a better alignment to the content standards to include standard 6.EE.6 in the Lesson Objectives and remove standards 6.EE.2b and 6.EE.9 as this lesson does not attempt to get at the depth of these standards. It is also recommended that adding instructional supports for addressing diverse learners such as ELLs (example; patterns, duplicates, Rails, rungs) or for students who finish quickly and accurately, would further enhance the lesson.

As the lesson is currently written, there is no opportunity for students to apply their mathematical understanding in an independent manner. The reviewers recommend the addition of some type of exit ticket or other opportunity for students to complete independently and submit to inform future instruction.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:

3: Meets most to all of the criteria in the dimension

2: Meets many of the criteria in the dimension

1: Meets some of the criteria in the dimension

0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:

E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV **(total 11 – 12)**

E/I: Exemplar *if* Improved – Aligned and needs some improvement in one or more dimensions **(total 8 – 10)**

R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions **(total 3 – 7)**

N: Not Ready to Review – Not aligned and does not meet criteria **(total 0 – 2)**

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:

3: **Exemplifies CCSS Quality -** meets the standard described by criteria in the dimension, as explained in criterion-based observations.

2: **Approaching CCSS Quality -** meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.

1: **Developing toward CCSS Quality -** needs significant revision, as suggested in criterion-based observations.

0: **Not representing CCSS Quality -** does not address the criteria in the dimension.

Descriptor for Overall Ratings:

E: **Exemplifies CCSS Quality –** Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.

E/I: **Approaching CCSS Quality –** Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.

R: **Developing toward CCSS Quality –** Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.

N: **Not representing CCSS Quality –** Not aligned and does not address criteria.