EQuIP Review Feedback

Lesson/Unit Name: Triangles To Order
Content Area: Mathematics
Grade Level: 7

Overall Rating:

E/I

Exemplar *if* Improved

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS: ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ❒ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	Triangles to Order is a 2-day lesson designed to help students discover which particular combinations of side and angle specifications will result in a unique triangle, several triangles, or no triangles, which is standard 7.G.A.2. The lesson also invites students to go on to generalize and develop an understanding about when a triangle can or can't be created and engages students in developing a conceptual understanding of the angle and side criteria of a triangle. The Standards for Mathematical Practice are intrinsic to the lesson, but they are not specifically identified for teachers and students who may not be familiar with these practices as the research-based habits of mathematically proficient students. There is evidence in this lesson that students need to make sense of problems and persevere in solving them (SMP.1) in the Workshop section, construct viable arguments and critique the reasoning of others (SMP.3) in the Post, Share, Comment section, and look for repeated reasoning (SMP.8) in the Focus Problem: Same Content in a New Context section. Students also need to be able to use appropriate tools strategically (ruler & protractor) as evidence of SMP.5. A strong suggestion for this dimension is for the author to document which Standards for Mathematical Practice are expected to surface during the lesson and how those are related to the specific content and pedagogy of the lesson. This is especially important for experienced teachers who are still refining their implementation of the Common Core Standards for Mathematics and for new teachers who may need to realize that there are two sets of standards - content standards and standards for mathematical practice. Those Standards for Mathematical Practice mentioned above may be good starting points, and the author may want to consider other practice standards that are encouraged during the lesson. There is a balance of mathematical procedures and deeper conceptual understanding in this lesson. There is evidence of this from the beginning of the lesson Launch that activates prior knowledge and builds conceptual understanding of triangles and the structured exploration of side and angle conditions throughout the lesson to determine if a unique triangle, more than one triangle, or no triangle can be made. Students are required to develop the skills of using a protractor and ruler to make their triangles. Although these skills are not a "mathematical procedure", they are essential for understanding the conditions needed to construct a triangle and the relationships inherent between the sides and angles of triangles. All of these are very important skills and conceptual understandings for the more complex studies of triangles in high school.
Rating: **2 – Meets many of the criteria in the dimension**	

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS: ❒ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades. ✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions. ✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following: – **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations. – **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding. – **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	FOCUS This lesson is developed for 7.G.A (Draw, Construct and describe geometrical figures and describe the relationship between them.) and specifically for standard 7.G.A.2 (Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.). The lesson is strongly focused on this standard, which is considered to be additional work for grade 7. The lesson does not have any documented connections to the major work of the grade in the RP (Ratios and Proportional Relationships), EE (Expressions and Equations), or NS (The Number System) domains. There is perhaps one missed opportunity on page 6 of the lesson, when reinforcing the 180 degree rule for a triangle, for students to go a little deeper with the conditions needed for similar triangles --- that is given three angles that add to 180 degrees, there are many triangles of different sizes, but they are all similar. This is an opportunity to connect to the Ratios and Proportional Relationships domain and the definition of similarity. The author might consider if there are any other missed opportunities to make logical (not contrived) connections to other major work in the NS or EE domains. COHERENCE Students in grade 5 classify two-dimensional figures in a hierarchy based on properties (5.G.B, which gives students a foundation for the generalizations and examination of specific criteria, such as angles and sides, expected in this lesson. In grade 6 students found the area of triangles. In 7th grade 7.G.A.1 asks students to solve problems involving scale drawings of geometric figures which includes computing actual lengths and from a scale drawing. As mentioned earlier, the concept of similar figures (those with the same angle measures) would make a nice within grade and across grade connection to RP standards from both 6th and 7th grade. In 8th grade, students use similar right triangles and proportional relationships to understand the concept of constant rate of change for a linear function. Progressing to high school, these are foundational concepts not only for continuing the study of linear functions, but also for the study of Similarity, Right Triangles, and Trigonometry (G-SRT) standards. The connections described above are not made apparent to teachers or students in the actual lesson. However, in the linked materials "Triangle Conventions and Mechanics" and "Triangles and Constraints" connections are made to the foundations for high school geometry that this 7.G.A.2 standard and lesson support. It is helpful for educators who might use the lesson to know just how important these linked resources are so that they have a clear understanding of ways to better connect student prior knowledge to the current concepts and also have students realize importance of these concepts to future mathematics work. RIGOR Application - Students actively engage in the use of angle and side "specifications" to draw triangles and determine if the specifications will or will not generate a triangle. However, in this lesson, there is no application to real world situations other than the brief look at triangles in graphic design during the Launch. The main emphasis of this lesson for 7.G.A.2 is the conceptual understanding and application of use in mathematical, not real-world context.

	Conceptual Understanding - The students work through a number of given specifications to eventually make generalizations related to determining a unique triangle, more than one triangle, or no triangle. The Pose a Problem Triangle Conditions handout cards provide a structure for the exploration by providing 12 side lengths in centimeters and 12 angle measures from which to choose to make a triangle. This structure allows enough room for students to pursue many combinations and then test them using scratch paper, rulers, and protractors, all the while comparing and contrasting combinations that worked with those that did not. Procedural skill and fluency - Students practice with the ruler and protractor as they construct their triangles with as much accuracy as possible. Many times students do not have the practice needed with these tools to become better with their use. They begin to generalize the rules for the conditions that will make one unique, many, or no triangles. Because of the above evidence, there is a balance of application, conceptual understanding and skill in this lesson, at least as much as is expected within the scope of 7.G.A.2.
Rating: **3 – Meets most to all of the criteria in the dimension**	

Dimension III – Instructional Supports

The lesson/unit is responsive to varied student learning needs: ✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media. ✓ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline. ✓ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking. ✓ Addresses instructional expectations and is easy to understand and use. ❒ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners. – Supports diverse cultural and linguistic backgrounds, interests and styles. – Provides extra supports for students working below grade level. – Provides extensions for students with high interest or working above grade level. <u>*A unit or longer lesson should:*</u> ❒ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including	This lesson, as does every Poster Problem lesson on the www.serpmedia.org website, has six phases in which student understanding of the content is developed. Each phase moves students toward developing an understanding of the conditions needed to make a triangle. It is an inquiry-based lesson that provides the resources for students to explore and make discoveries, but within the structure of given conditions and product expectations. The introduction/overview for the teachers on page 1 provides background information and a general direction and purpose of the lesson. The learning objectives are clearly identified and correlated with specific standards. The "Challenges" identify some of the problems that students might encounter during the lesson. The Teacher Tune Up section provides links to support documents which clearly help any educator refresh or learn the critical details and content understanding so that they can lead the students towards mastery of the standard, no matter their entry point. Following are the instructional supports evidenced in the flow of the lesson structures. 1. Launch (p.2) - This portion of the lesson includes an example of why triangles are important, making the connection to their use in computer graphics. The second slide requires students to describe the attributes of a right triangle with accurate geometric terms. A second possible activity is posed in which a "teller" must verbally describe a triangle to a "drawer", requiring both the "teller" and the "drawer" to understand the vocabulary of triangles. The directions seem to be clear enough for most educators to follow. It is important, though, to maintain the precise vocabulary. Students can and will use the appropriate vocabulary (i.e. adjacent, congruent, line segment, angle, opposite, perpendicular) when used correctly and consistently. NOTE: On Slide 2 there are inconsistencies/errors in the text descriptions of the vertices and side lengths and those actually shown on the slide.

models, using a range of questions, checking for understanding, flexible grouping, pair-share). ❒ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently. ❒ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ❒ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	2. Pose A Problem (p.3) - Students have an opportunity to work in small groups to share ideas and determine which side and angle "specifications" will generate a triangle. The Triangles Conditions handout provides 12 side length cards and 12 angle measure cards that students cut out, mix up, and randomly place in one stack upside down. 3 cards are randomly drawn from the stack and placed on a provided template/form as a type of "hypothesis" for the conditions to generate a triangle. Generating and testing hypotheses is a research-based instructional strategy that can have a high effect size for learning. Students then attempt to construct the triangle with the given conditions. They record their findings on the provided Triangles Record Sheet handout. The general directions are clear and provide opportunities for differentiation for students. The note to the educator regarding when to end the activity is a good reminder, that the time and examples students need to get a handle on the concept may vary and needs to be determined by the educator. 3. Workshop (p.4) - Students embark on a somewhat inquiry-based look at more "specifications" and figure out if any "unique" triangles can be made that can be made with those specifications. They create a two-column poster to organize their findings regarding when a triangle can be made and when it cannot. They are also asked to generalize based upon their findings. This can very well serve to elicit a variety of student understandings. There are specific suggestions on the bottom of p.4 for the teacher to help students if they are not getting to the "two" and "many" examples. The student directions on the Workshop and Poster Handout for the Workshop are quite lengthy and will probably need some further clarification and support from the teacher than that provided on p.4. Suggestions also include providing more probing questions, and possible misconceptions for teachers to use during the Workshop portion of the lesson. As written, this Poster Problem is fine for teachers experienced with the workshop model of teaching, but new teachers and direct-instruction teachers may really struggle. Directions should also include suggestions for how to develop a student mind-set that mistakes are a natural part of learning and that products can be made better through editing and revision. 4. Post, Share, Comment (p.5) - All groups share their poster solutions and groups have an opportunity to comment on others' solutions. During this time the teacher determines which ones to highlight. The examples provided may not resemble the ones that the students actually generate, but the examples and the explanations of the example posters, could be a major benefit for the teacher to know what to eventually elicit from the students. It might be helpful if students had a rubric to evaluate their posters for the directions provided on the Workshop and Poster handout before posting them. 5. Strategic Teacher-Led Discussion (p.6) - A whole group discussion is led by the teacher who addresses the learning objectives in the lesson and the comments students have made on each other's posters. This section has numerous directions and supports for the teacher to help students understand the desired content, based upon five different levels of

	mastery. These supports could be used even if students do not actually come up with the ideas during the poster creations as the teacher could use these guides to help move students forward. The sections on Supporting Generalization, Connecting across Groups, and Three angles: the miracle of 180 degrees provides good guidance for the educator to help draw students to the desired understandings. 6. Focus Problem (p.7): Same Concept in a New Context - Students work on a focus problem that addresses the same concepts as those addressed in the lesson. The directions help the student understand the expectations. What is not clear is if this problem is meant to be completed in a group, pair, or independently. It is also not clear as to the intent - is this to be used as a formative or summative assessment? Overall, this lesson provides access to varied student needs and provides some support for the teacher by: - Setting the expectation for accurate vocabulary in the Launch and establishing foundational information regarding specifications of triangles. - Engaging students in productive struggle through challenging exploratory activities that engage the students in constructing triangles given certain conditions by using a protractor and a ruler. - Providing a scaffolded sequence of learning by strategically moving from the analysis of the specifications/conditions of triangles in the Launch and Pose a Problem portion of the lesson to the analysis of and generalizations during the Workshop and Strategic Teacher-Led Discussion portions of the lesson. - Providing natural differentiation by allowing students to choose how to explore and discover the specifications/conditions for the triangle creations. The author might consider adding some instructional supports for addressing ELLs and other diverse learners. - All materials, including handouts, Teacher Tune Ups and the actual lessons for teachers are easily downloaded from the www.math.SERPmedia.org website.
Rating: **3 – Meets most to all of the criteria in the dimension**	

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills: ❒ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. *A unit or longer lesson should:*	Most of the assessment in this two-day lesson is formative, through teacher observations and the use of prompts to help students move toward an understanding of the specifications/conditions for triangles. This lesson is posted on the www.math.SERPmedia.org website as "diagnostic", however the downloaded PDF version does not make that clear to teachers. The formative methods and activities presented provide many opportunities for the teacher to informally assess student understanding of the concepts required for mastery of 7.G.A.2. Answer keys are provided for handouts that provide sufficient guidance for expected student performance. At the end of the lesson, the Focus Problem does provide an opportunity for students to individually and independently apply their learnings from the lesson. Through this problem, teachers can gauge just how well each student understands how to construct triangles according to specifications, recognize that with three conditions you can often - but not always-

✓ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.	determine a triangle, and if students understand the rules for when three conditions determine a triangle as identified in the learning objectives. The author might want to consider providing different ways the Focus Problem might be used for assessment of individual learning or providing another option for assessment. Lastly, a writing prompt or exit ticket might be considered at the end of Day 1 for the teacher to gauge student understanding at that point in the lesson sequence.
Rating: **2 – Meets many of the criteria in the dimension**	

Summary Comments

Overall this lesson meets the needs of students to reach mastery of the concepts expected in 7.G.A.2. It provides teachers with a quality inquiry-based lesson to stimulate student interest in and exploration of the specifications needed for constructing triangles. There are opportunities in the lesson to support students who need more help and also for students who already grasp the concepts to move ahead to more complex topics through self-motivation and exploration. The exploratory work during the Workshop provides a bridge for students to connect conceptual understanding to making generalizations about the three conditions which can lead to determine a unique triangle, more than one triangle, or no triangle.

Following is a summary of suggestions for improvement that may be considered. Details are included in the previous separate dimension sections of this review.
- Include connections between the student expectations for engaging in the content and the related Standards for Mathematical Practice.
- Remind teachers of the connected standards and knowledge from grades 5 and 6, as well as the within grade connections to grade 7 major work of the grade, and also the coherence with grade 8 and high school mathematics standards that will follow.
- Provide clearer directions, probing questions, and possible misconceptions, for teachers to address during the Workshop portion of the lesson. As written, this Poster Problem is fine for teachers experienced with the workshop model of teaching, but new teachers and direct-instruction teachers may really struggle. Directions should also include suggestions for how to develop a student mind-set that mistakes are a natural part of learning and that products can be made better through editing and revision.
- Add instructional supports for addressing ELLs and other diverse learners.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:

3: Meets most to all of the criteria in the dimension
2: Meets many of the criteria in the dimension
1: Meets some of the criteria in the dimension
0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:

E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV **(total 11 – 12)**
E/I: Exemplar *if* Improved – Aligned and needs some improvement in one or more dimensions **(total 8 – 10)**
R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions **(total 3 – 7)**
N: Not Ready to Review – Not aligned and does not meet criteria **(total 0 – 2)**

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:

3: **Exemplifies CCSS Quality -** meets the standard described by criteria in the dimension, as explained in criterion-based observations.
2: **Approaching CCSS Quality -** meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
1: **Developing toward CCSS Quality -** needs significant revision, as suggested in criterion-based observations.
0: **Not representing CCSS Quality -** does not address the criteria in the dimension.

Descriptor for Overall Ratings:

E: Exemplifies CCSS Quality – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
E/I: Approaching CCSS Quality – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.

R: **Developing toward CCSS Quality –** Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.

N: **Not representing CCSS Quality** – Not aligned and does not address criteria.