Lesson/Unit Name: Two-Digit Array Multiplication  
Content Area: Mathematics  
Grade Level: 4  

**Dimension I – Alignment to the Depth of the CCSS**

**The lesson/unit aligns with the letter and spirit of the CCSS:**

- Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
- Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

This lesson addresses half of the standard for grade four - 4.NBT.B.5. It does provide a depth of understanding for teaching multiplying a 2-digit number by a 2-digit number using arrays. The lesson states that it will take one hour to present, yet further in the lesson, the writer states that the lesson may take more time for students to practice the procedure. Additional time might be considered in the Overview box by C-PALMS, so teachers understand that it is okay to take the time to go deeper.

The SMPs addressed in this lesson are #7 and #3. However, SMP 7 was weaker in its alignment, especially in critiquing the work of others. SMP 6 was not listed; however, the reviewers felt the lesson did a good job with this SMP especially with precise academic language. The author included these and not the C-PALMS group. The SMPs are critical in helping to get to the rigor for the lesson. A note to teachers in the specific area where those SMPs are located will help teachers connect to the content to inform their understanding of the standard.

The balance between the procedure and conceptual understanding is a strength of this lesson. The teacher helps students get to place value understanding in order to build the procedure of multiplication of numbers greater than one digit. This lesson will lead students to an efficient algorithm for multiplication.

*Rating: 3 – Meets most to all of the criteria in the dimension*

**Dimension II – Key Shifts the CCSS**

**The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:**

- **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- **Rigor:** Requires students to engage with and demonstrate challenging mathematics with

This lesson does target a major standard for 4th grade. The depth that the lesson goes to with tying the place value build first to the algorithm of partial products is especially well done. The students are held accountable for only the work at their grade level.

Coherence is addressed in the section titled Prior Knowledge. The standards are tied to grade three through 3.OA.B.5 and 3.NBT.A.3. There is no scaffold for a tie to grade 5. A suggestion for C-PALMS would be to add a section for writers to include coherence in the Overview box.

This lesson allows and balances application, conceptual understanding, and fluency. There are opportunities for students to speak about their understanding as they explain their solutions, if the teacher chooses to allow students to do the explaining. The option for the teacher to explain is stated, which would lower the rigor of the lesson and the ability for the teacher to assess understanding. It might be helpful to provide some strategies for selecting students to explain their solutions. Students are also asked to explain to the teacher as they complete the practice problems. However, they are not asked to write about their understanding, other than

*Overall Rating: E – Exemplar*
appropriate balance among the following:

- **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
- **Conceptual Understanding:** Develops students’ conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
- **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

A unit or longer lesson should:

- **…”**

The writer includes clear directions for teaching the lesson along with guidance through the use of actual student examples (see an example; Array Picture 1-4, etc.). The writer also warns teachers: "The Independent Practice may need to be completed on another day." This helps encourage teachers to slow down to make sure students understand the concept. Also included was the use of a segment from The Teaching Channel on "My Favorite No" to help teachers see and hear how to use this as a formative assessment. Document cameras or overhead projectors are the technology component of the lesson.

The use of vocabulary especially in relationship to place value is an important part of the lesson. The writer states, "Do not allow short cuts in vocabulary...." Also, the writer states, "As students begin to explain their strategies, encourage complete and specific explanations." Concrete representations are built through the use of base 10 blocks. Students are encouraged to move to pictures, "As students show they understand, you can let them draw the pieces rather than use the blocks." The use of equations is built into the concept after the picture is made, and the discussion occurs as to what each part of the product represents.

The writer states, "Let the children struggle, talk with one another, and create some possibilities. You will need to assess the frustration in the room. Offer hints or stop the exploration and discuss as a class, if you feel the students are too confused." Students are asked to be able to explain what each part of the array represents, why you add the partial products and not multiply, and compare/contrast arrays that have no numbers but physical models. It would be helpful to describe specific student behaviors that indicate "too confused" in order to help teachers unaccustomed to allowing students to struggle not to stop too early. Also, a more thorough discussion of how the guiding questions might be used during each stage of the lesson...
approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).

- Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Aspects needed in a quality lesson are used - whole group and individual practice. The one suggestion to make here is the idea that the students present their arrays OR the teacher shows; default to the students sharing so the teacher will know what a student understands and is able to do.

The differentiation is listed in the section titled Accommodations and Recommendations. Students who may be struggling, those who need EL help and Extensions are given with a brief explanation of what to provide for each.

Rating: 3 – Meets most to all of the criteria in the dimension

**Dimension IV – Assessment**

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:

- Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

The assessments with this lesson are designed to elicit direct, observable evidence of the student independently mastering the targeted CCSS. This lesson includes a pre-assessment (labeled as a formative assessment) to assess prior knowledge. Clear directions are given for "My Favorite No" as well as sample student responses. Giving visual samples of work is a strength of this lesson. When teachers can visualize what the answers should look like it helps them check their own understanding of the material. One suggestion would be to include an estimate of the amount of time the teacher should spend on this section of the lesson. Problems are included for the independent practice and the summative assessment. Again, the writer encourages teachers to change the names to increase student motivation.

The assessments in this lesson are accessible and unbiased. Grade-level language is used by the teacher, and it is expected students will also use grade-level mathematics language to show understanding of the standard. By stating this expectation, the writer is getting students to understand the depth of the standard.

Answer keys are included for the independent practice and the summative assessment. A suggestion for improvement would be to include the arrays on the answer sheets instead of just the answer to the multiplication problems. This would be beneficial for a new or struggling elementary math teacher. Some of the answer sheets have the answers in red; it would be beneficial if all the answer sheets did this, as well.

During part of the formative assessment the teacher is checking students' work during the independent practice, it might save the teacher "racing around the room" if students had the opportunity to self or peer-assess during this phase of instruction.

Rating: 3 – Meets most to all of the criteria in the dimension

**Summary Comments**

This lesson is an example for writing individual lessons for the CCSS. The lesson has a specific standard it targets; within that standard it is narrowed for the lesson to allow for procedural fluency for the strategy of using arrays to build an area model.
Vocabulary is used expertly to help students understand conceptually the standard for multiplying two-digit numbers by a two-digit number. If used by teachers with a limited understanding and experience in promoting productive struggle in students, additional guidance might be needed. C-PALMS might want to consider including in their Overview box the specified vocabulary contained within this lesson. Are Key Words used as a search engine to find lessons or is "key words" used as a term for vocabulary from the lesson that teachers should focus on? (The assumption by the reviewers is that Key Words are being used as a search engine.) Assessment focuses not only on the array being used to find the answer, but on the real-world context for the use of multiplication by students.

Rating Scales
Rating Scale for Dimensions I, II, III, IV:
3: Meets most to all of the criteria in the dimension
2: Meets many of the criteria in the dimension
1: Meets some of the criteria in the dimension
0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:
E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (total 11 – 12)
E/I: Exemplar if Improved – Aligned and needs some improvement in one or more dimensions (total 8 – 10)
R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (total 3 – 7)
N: Not Ready to Review – Not aligned and does not meet criteria (total 0 – 2)

Rating Descriptors
Descriptors for Dimensions I, II, III, IV:
3: Exemplifies CCSS Quality - meets the standard described by criteria in the dimension, as explained in criterion-based observations.
2: Approaching CCSS Quality - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
1: Developing toward CCSS Quality - needs significant revision, as suggested in criterion-based observations.
0: Not representing CCSS Quality - does not address the criteria in the dimension.

Descriptor for Overall Ratings:
E: Exemplifies CCSS Quality – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
E/I: Approaching CCSS Quality – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
R: Developing toward CCSS Quality – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.
N: Not representing CCSS Quality – Not aligned and does not address criteria.