Achieve, Inc.

MAP Algebra Expectations

Algebra

K.9 Identify, sort and classify objects.

K.9a Sort and classify objects by attribute and identify objects that do not belong in a particular group.

- Recognize attributes that involve colors, shapes (e.g., triangles, squares, rectangles, and circles), and patterns (e.g., repeated pairs, bilateral symmetry).

  Example: Identify the common attribute of square in a square book, square table, and square window.

  Example: Distinguish different patterns in ABABABA, ♦♥♥♦♥♥♦♥♥.

K.9c Recognize related addition and subtraction facts.

- Use objects to demonstrate "related facts" such as $7 - 4 = 3, 3 + 4 = 7, 7 - 3 = 4$.

Algebra

1.8 Recognize and extend simple patterns

1.8a Skip count by 2s and 5s, and count backwards from 10.

1.8b Identify and explain simple repeating patterns

- Find repeating patterns in the discrete number line, in the 12 x 12 addition table, and in the hundreds table (a 10 x 10 square with numbers arranged from 1 to 100).

  Note: Use examples based on linear growth (e.g., height, age).

- Create and observe numerical patterns on a calculator by repeatedly adding or subtracting the same number from some starting number.

1.8c Determine a plausible next term in a given sequence, and give a reason.

Note: Without explicit rules, many answers to "next term" problems may be reasonable. So whenever possible, rules for determining the next term should be accurately described. Patterns drawn from number and geometry generally have clear rules; patterns observed in collected data generally do not.

1.9 Find unknowns in problems involving addition and subtraction.

1.9a Understand that addition can be done in any order but that subtraction can not.

- Demonstrate using objects that the order in which things are added does not change the total, but that the order in which things are subtracted does matter.
- Use the fact that $a + b = b + a$ to simplify addition problems.

  Examples: $2+13=13+2=15$ (by adding on);
  $7+8+3=7+3+8=10+8=18$.

  Note: The relation $a + b = b + a$ is known as the *commutative* property of addition. It reduces significantly the number of addition facts that need to be learned. However, the vocabulary is not needed until later grades.

- Demonstrate understanding of the basic formula $a + b = c$ by using objects to illustrate all eight number sentences associated with any particular sum:

  Example: 8 + 6 = 14, 6 + 8 = 14; 14 = 8 + 6, 14 = 6 + 8;
  14 – 8 = 6, 6 = 14 – 8; 14 – 6 = 8, 8 = 14 – 6.

1.10 Understand how adding and subtracting are inverse operations.

1.10a Demonstrate using objects that subtraction undoes addition, and *vice versa*.

- Subtracting a number undoes the effect of adding that number, thus restoring the original. Similarly, adding a number undoes the action of subtracting that number.

  Example: 2 + 3 = 5 implies 5 - 2 = 3 and *5 - 2 = 3* implies *2 +3 = 5*.

- Use the inverse relation between addition and subtraction to check arithmetic calculations.

  Note: Addition and subtraction are said to be *inverse* operations because subtraction undoes addition and addition undoes subtraction. However, this vocabulary is not needed until later grades.

  ***Caution*:** Subtraction is sometimes said to be equivalent to "adding the opposite," meaning that 5 - 3 is the same as 5 + -3. Here the "opposite" of a number is intended to mean the negative of a number. However, since negative numbers are not introduced until later grades, this formulation of the relation between addition and subtraction should be postponed.

Algebra

2.11 Create, identify, describe, and extend patterns.

2.11a Fill in tables based on stated rules to reveal patterns.

- Find patterns in both arithmetic and geometric contexts.

2.11b Record and study patterns in lists of numbers created by repeated addition or subtraction.

- Create patterns mentally (by counting up and down), by hand (with paper and pencil), and by repeated action on a calculator.

  Examples: 3, 8, 13, 18, 23, ...; 50, 46, 42, 38, 34, ...

2.12 Find unknowns in simple arithmetic problems

2.12a Solve equations and problems involving addition, subtraction and multiplication with the unknown in any position.

Note: In the early grades it is better to signify the unknown with a symbol such as [ ], ?, or ♦ that carries the connotation of unknown rather than with an alphabetic letter such as *x.*

2.12b Understand and use the facts that addition and multiplication are commutative and associative.

- Use parentheses to clarify groupings and order of operation.
- Recognize terms such as *commutative* and *associative.*

  Note: It is not necessary for children at this grade to use or write these words, merely to recognize them orally and to know the properties to which they refer.

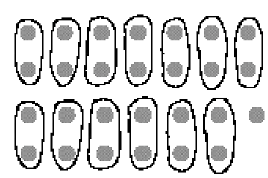
2.12c Recognize how multiplication and division are, like addition and subtraction, inverse operations.

2.13 Understand basic properties of odd and even numbers.

2.13a Explain why the sum of two even numbers is even, and that the sum of two odd numbers is also even.

- Use diagrams to represent even and odd numbers and to explain their behavior.

  Example: The representation at the right shows that 14 is even and 13 is odd.



2.13b Answer similar questions about subtraction and multiplication of odd or even numbers.

Algebra

3.12 Explore and understand arithmetic relationships among positive whole numbers.

3.12a Understand the inverse relationships between addition and subtraction and between multiplication and division, and the commutative laws of multiplication and addition.

- Show that subtraction and division is not commutative.

3.12b Find the unknown in simple equations that involve one or more of the four arithmetic operations.

> ***Note:*** To emphasize the process of solving for an unknown, limit coefficients and solutions to small positive whole numbers.
>
> ***Examples:*** $3 \times ? = 3 + 6$; $? \div 5 = 5 \times 55$; $36 = ? \times ?$.

3.12c Create, describe, explain, and extend patterns based on numbers, operations, geometric objects, and relationships.

- Explore both arithmetic (constant difference) and geometric (constant multiple) sequences.

  Examples: 100, 93, 86, 79, 72, ...; 2, 4, 8, 16, ...; 3, 9, 27, 81, ...

- Understand that patterns do not imply rules; rules imply patterns.

Algebra

4.12 Use properties of arithmetic to solve simple problems.

4.12a Understand and use the commutative, associative, and distributive properties of numbers.

- Use these terms appropriately in oral descriptions of mathematical reasoning.
- Use parentheses to illustrate and clarify these properties.

4.12b Find the unknown in simple linear equations.

- Use a mixture of whole numbers, fractions, and mixed numbers as coefficients.

Examples: 24 + n = n -2; 3/4 + p = 5/4 - p

Note: "Simple" equations for Grade 4 are those that require only addition or subtraction (e.g., 3/4 + [ ] = 7/4) or a single division whose answer is a whole number (e.g., 3 x [ ] = 12).

Note: There is no need to use the term *linear* since these are the only kinds of equations encountered in Grade 4.

4.13 Evaluate simple expressions

4.13a Find the value of expressions such as $na + b$ and $na - b$ where a, b, and n are whole numbers or fractions and where $na \geq b$.

- Make tables and graphs to display the results of evaluating expressions for different values of *n* such as n = 1, 2, 3, ...

Note: Evaluating an expression involves two distinct steps: substituting specific values for letter variables in the expression, and then carrying out the arithmetic operations implied by the expression. Working with expressions both introduces the processes of algebra and also reinforces skills in arithmetic.

Note: Avoid negative numbers since systematic treatment of operations on negative numbers is not introduced until grade 6.

4.13b Evaluate expressions such as $\frac{a}{b}+\frac{c}{nb}$, where *a, b, c*, and *n* are whole numbers.

4.13c Evaluate expressions such as $\frac{1}{a}+\frac{1}{b}$ where *a* and *b* are single digit whole numbers.

Example: The value of $\frac{a}{b}+\frac{c}{nb}$ when a =1, b= 2, c = 3, and n = 4 is ${}^{1}/_{2} + {}^{3}/_{8} = {}^{4}/_{8} + {}^{3}/_{8} = {}^{7}/_{8}$.

Note: Addition of fractions is limited to cases included in the Grade 4 expectations, namely, unit fractions with denominators under 10 and other fractions where one denominator is a multiple of the other.

Algebra

5.16 Find the unknown in simple linear equations.

5.16a Equations that require only simple calculation should be solved mentally (that is, "by inspection"):

$96 + 67 = b + 67$	$^{3}/_{4} + {}^{5}/_{8} - {}^{5}/_{8} = p$	$a + {}^{3}/_{5} = {}^{3}/_{5}$
$39 - k = 39 - 40$	$^{3}/_{5} - {}^{3}/_{8} + {}^{5}/_{8} = d + {}^{3}/_{8} - {}^{5}/_{8}$	$^{1}/_{5} + {}^{2}/_{5} = b + {}^{6}/_{5}$
$78 + b = 57 + 79$	$53 + 76 = 51 + 76 + d$	

5.17 Evaluate and represent simple expressions.

5.17a Translate between simple expressions, tables of data, and graphs in the coordinate plane.

5.17b Understand and use the conventions for order of operations (including powers).

Example: $ax^2+bx = (a(x^2))+(bx)$, not $(ax)^2+bx$.

5.17c Evaluate expressions such as

- *nr* where *n* is a whole number and *r* is a fraction.
- *nab/(na-b)*when n, *a* and *b* are whole numbers and where n*a* > *b*.
- $^{a}/_{b} + {}^{c}/_{d}$,where *a*, *b, c, d* are positive whole numbers.
- 1/*ab* where *a* and *b* are positive whole numbers.
- *a*/*b* where one of *a* or *b* is a positive whole numbers and one is a fraction.

Note: Avoid expressions that introduce negative numbers since systematic treatment of operations on negative numbers is not introduced until grade 6.

Note: Working with expressions both introduces the processes of algebra and also reinforces skills in arithmetic.

5.17d Understand the importance of not dividing by zero.

Algebra

6.11 Understand that the system of negative and positive numbers obeys and extends the laws governing positive numbers.

6.11a The sum and product of two numbers, whether positive or negative, integer or fraction, satisfies the commutative, associative and distributive laws.

- For any numbers a, b, c, (whether positive, negative, or zero),
  $a + b = b + a$, $a \times b = b \times a$ (commutative);
  $a + (b + c) = (a + b) + c$; $a \times (b \times c) = (a \times b) \times c$ (associative);
  $a \times (b + c) = (a \times b) + (a \times c)$ (distributive).

  Example: $(-3) \times 5 = -(3 \times 5)$, because $((-3) \times 5) + (3 \times 5) = ((-3) + 3) \times 5) = 0 \times 5 = 0$. Hence the sum of $(-3) \times 5$ and 3×5 is 0, so $(-3) \times 5 = -(3 \times 5)$.

  Note: This example can usefully be demonstrated on the number line in a way that avoids the formality of parentheses required above.

6.11b Understand why the product of two negative numbers must be positive.

- Since a negative number -a is defined by the equation -a + a = 0, the distributive law forces the product of two negative numbers to be positive.

  Example: To show that $(-3) \times (-5) = 3 \times 5$, we demonstrate that the sum of the left side $[(-3) \times (-5)]$ with the negative of the right $[3 \times 5]$ is zero:

  $$((-3) \times (-5)) + (-(3 \times 5)) = ((-3) \times (-5)) + ((-3) \times 5) = (-3) \times ((-5) + 5) = (-3) \times 0 = 0.$$

  The key middle step uses the distributive law.

6.11c Understand why the quotient of two negative numbers must be positive.

- Division is the same as multiplication by a reciprocal. If a number b is negative, so is its reciprocal 1/b. So if a and b are both negative, a/b is positive since it equals the product of two negative numbers: a x (1/b).

  Example: If p = -12/-3, then $-12 = p \times (-3)$. Since 4 x-3 = -12, this yields p = 4, hence -12/-3 = 12/3 = 4.

6.12 Represent and use algebraic relationships in a variety of ways.

6.12a Recognize and observe notational conventions in algebraic expressions.

- Understand and use letters to stand for numbers.
- Recognize the use of juxtaposition (e.g., $3x$, ab) to stand for multiplication, and the convention in these cases of writing numbers before letters.
- Recognize the tradition of using certain letters in particular contexts.

  Note: Most common: *k* for constant, *n* for whole number, *t* for time, early letters (a, b, c) for parameters, late letters (u, v, x, y, z) for unknowns.
- Recognize different conventions used in calculator and computer spreadsheets (e.g., * for multiplication, ^ for power).
- Understand and use conventions concerning order of operations and use parentheses to specify order when necessary.

  Note: By convention, powers are calculated before multiplication (or division), and multiplication is done before addition (or subtraction).

  Example: $3x^2$ means $3(x•x)$, not $(3x)•(3x)$; $3x^2-7x$ means $(3x^2) - (7x)$, not $(3x^2-7)•x$.
- Evaluate expressions involving all five arithmetic operations (addition, subtraction, multiplication, division, and power).

Note: For the most part this is review. What is new is the introduction of powers and the shift to use of letters (rather than boxes, question marks and other placeholders) to represent generic or unknown numbers.
Examples: $3x^2-7x+2$, when $x = 3$ or 1/3; $2x^{-3}+x$, when $x = 2$, or 1/2, or 0, or -3); $3x^2-2xy$, *when* $x = 2$ *and* $y = 6$.

6.12b Solve problems involving translation among and between verbal, tabular, numerical, algebraic, and graphical expressions.

- Write an equation that generates a given table of values
  Note: Limit examples to linear relationships with integer domains.
- Graph ordered pairs of integers on a coordinate grid.
  Example*:* Prepare *scatterplots* of related data such as students' height vs arm length in inches.
- Generate data and graph relationships concerning measurement of length, area, volume, weight, time, temperature, money, and information.
- Understand why formulas or words can *represent* relationships whereas tables and graphs can generally only *suggest* relationships.
  Note: Unless the rule for a table (or graph) is specified (e.g., in a tax table), the numbers themselves cannot determine a relationship that extends to numbers not in the table.
  Note: The issue here is about the lack of precision inherent in a graph, not about the possibility, which is present in some graphs, of ambiguity concerning which of two very different points on the graph are associated with a given input value. The common example of ambiguity concerns the graph of the equation for a circle, but such graphs are not among those studied in grades K-8.

Algebra

7.9 Understand the properties of linear functions and use these properties in a variety of applications.

7.9a Understand the concept of a function as a rule that assigns one number (the output) to another number (the input).

- The notation f(x) represents the number that the function f assigns to the input x.
- Make tables of inputs *x* and outputs *f(x)* for a variety of rules that take numbers as inputs and produce numbers as outputs.
- Plot points on a coordinate plane to represent tables of function values.
- Understand that the *graph* of a function f is the set of points in the coordinate plane representing the ordered pairs (x, f(x)).

    Note: However, do *not* define a function as a set of ordered pairs.
- Use simple algebraic expressions to define rules for functions to be calculated, tabulated, and graphed.

    Note: While functions are often defined by formulas, they can be defined by other sorts of rules.

    Note: A ubiquitous source of function rules can be found on the function feature of spreadsheet software. Most of these functions are too advanced for middle grades, but many are not. They provide good sources of examples.

7.9b A function exhibiting a constant rate of change is called a *linear* function.

- A *constant rate of change* means that when the input changes at a constant rate, so does the output.

    Examples: f(x) = 2x; f(x) = 5-3x; f(side of square) = perimeter of square.

7.9c A linear function in which f(0) = 0 is called a *proportional relationship*.

- A proportional relationship can be represented by f(x) = kx where k is called the *constant of proportionality*.
- The graph of a proportional relationship f(x) = kx is a line with slope k that passes through the point (0, 0).

    Note: Similar triangles show that the point(x, kx) for any number x is on the line that connects (0,0) to (1,k).

7.9d A linear function can be represented by the function $f(x) = mx + b$.

- If f(x) = mx + b, then the function g(x) = f(x) - b =mx is a proportional relationship.

    Note: If $f(x)$ is a linear function, then $g(x) = f(x) - f(0)$ will be a proportional relationship (since $g(0) = 0$). That means that $g(x) = kx$, so $f(x) = g(x) + f(0) = kx + f(0)$. Setting $b= f(0)$ yields $f(x) = kx + b$.

    Note: By tradition, the constant of proportionality in a proportional relationship is usually denoted by *k*, while the similar constant in a linear function is denoted by the letter *m*.

    Note: The function $f(x) = mx + b$ is often written as $y = mx + b$ since when f(x) is graphed on a coordinate grid the values of f(x) are marked off on the vertical y-axis.

7.9e The graph of a linear function $f(x) = mx + b$ is a straight line.

Note: The graph of $f(x) = mx + b$ is the graph of the proportional relationship $g(x) = mx$ shifted up (or down) by *b* units. Since the graph of $g(x)$ is a straight line, so is the graph of $f(x)$.

- In the graph of $f(x) = mx + b$, m is the slope of the line and b is the y-coordinate of the point where the graph crosses the y-axis (i.e., the value of $f(x)$ when $x = 0$).

  Note: Slopes provide an algebraic way to show that the graph of $f(x) = mx + b$ is a straight line. Consider the slopes determined by any two points on the graph of $f(x)$. Since $f(x_1) = mx_1 + b$ and $f(x_2) = mx_2 + b$, it follows that $f(x_1) - f(x_2) = m(x_1 - x_2)$. Hence $m = (f(x_1) - f(x_2)) / (x_1 - x_2)$ is the same for any two points on the graph of $f(x)$.

  Example: Compare and contrast the graphs of $x = k$, $y = k$, and $y = kx$, where k is a constant.

7.9f Translate common linear phenomena into symbolic statements and interpret m and b in $f(x) = mx + b$ in terms of the original situation.

- Work fluently with directly proportional relationships and linear functions.
- Recognize contextual situations in which linear models are appropriate.
- Solve problems involving linear phenomena.

  Note: Common examples of linear phenomena include distance and time under constant speed; shipping costs under constant incremental cost per pound; conversion of measurement units (pounds and kilograms, degrees Celsius and degrees Fahrenheit); cost of gas in relation to gallons used; the height and weight of a stack of chairs.

7.10 Plot and interpret graphs of functions representing different non-linear relationships.

7.10a Create tables of coordinate points and plot graphs of various functions that are not linear.

- Plot points of non-linear functions by hand to gain experience with the coordinate system.

  Note: To understand what kinds of functions are linear it is important for students to work with examples of functions that are not linear.

  Examples: $f(x) = 3x^2 + 1$, $f(x) = 2x^3$, and $f(x) = 5/x$.

7.10b An *inversely proportional* relationship is represented by $f(x) = k/x$ where k is some non-zero number.

- The graph of $f(x) = k/x$ is a curve, not a line, and does not cross either the x nor the y-axis.

  Note: There is no value of x for which $f(x) = 0$, nor is there any value for $f(x)$ if $x = 0$.

  Note: To more sharply contrast a proportional relationship from an inversely proportional relationship, the former is often called a *directly proportional* relationship.

- Recognize quantities that are inversely proportional, for example, the relationship between lengths of the base and side of a rectangle with fixed area.

7.10c A *quadratic* function is represented by an expression of the form $f(x) = ax^2 + bx + c$ where a, b, and c are specific numbers, and x represents the input into the function.

- Except when $a = 0$, the graph of $f(x) = ax^2 + bx + c$ is a curve that always crosses the y axis but may or may not cross the x-axis.

  Note: When $x = 0$, $f(x) = c$, so the graph of $f(x)$ crosses the y-axis at the point $(0, c)$. When $a = 0$, $f(x) = bx + c$ which is a linear function.

- Quadratic functions may also appear in *factored form*: f(x) = (x - r)(x - s).
  Note: In this form, *f(x)*= 0 whenever *x = r* or *x = s*. So the graph of the function *f(x)* crosses the x axis at these two points.
- Recognize quantities that are represented by quadratic functions, for example, the relationship between lengths of the side of a square and its area.

7.10d Recognize whether information given in a table, graph, or formula suggests a direct proportional, linear, quadratic, inversely proportional, or other nonlinear relationship.

- Be able to identify graphs of simple linear and quadratic functions.

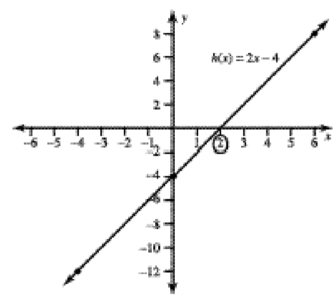
7.10e Solve simple problems involving relationships that are directly proportional, linear, quadratic, or inversely proportional.

7.11 Understand the concept of equation and the relation among equations, functions, and graphs.

7.11a Solve linear equations by graphing.

- If f(x) and g(x) are functions, the expression f(x) = g(x) is called an *equation*. *Solving an equation* means finding all value of x for which the equation is true.
- A common special case is when g(x) = 0. In this case solving the equation f(x) =0 means finding all value of x for which f(x) = 0.
- The solution to the equation f(x) = 0 are the values of x where the graph of the function f(x) crosses the x axis.

Example. As indicated by the illustration on the right, the graph of the linear function *f*(*x*) = 2*x* −4 crosses the x-axis at x = 2. This is the solution to the equation f(x) = 0, since f(2) = 2•2 - 4 = 0.



Note: Since in the coordinate plane the values of f(x) are plotted in the vertical direction along the y-axis, expressions such as $y = 7x + 5$ or $y = 3x^2 - 15x$ are often called equations. Graphing such an equation means graphing the function f(x) = 7x + 5 or $f(x) = 3x^2 - 15x$. Solving such an equation means finding the value of x for which y, that is, f(x) equals 0.

7.11b Solve linear equations by algebraic simplification.

- Use established properties of numbers to simplify expressions associated with linear functions.
- Add, subtract, and multiply simple polynomial expressions.

Example: (2*x*+5)+ (3-2*x*) = 2*x* + 5 + 3 - 2*x* = 8
(2*x*+5) - (3-2*x*) = 2*x* + 5 - 3 + 2*x* = 2 + 4*x*
$(2x+5) \times (3-2x) = 6x + 15 - 4x^2 - 10x = 15 - 4x - 4x^2$

- Recognize and use the fact that equals added to equals are equal and that equals multiplied by equals are equal.

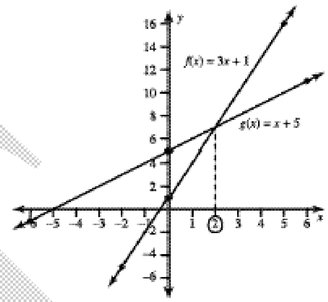
Note: More formally, if A= B and C= D, then A+B = C+D and AC=BD. These rules apply to polynomial expressions since each such expression represents a number, and the rules apply to numbers.

Caution: Be alert to anomalies caused by dividing by 0 (which is, of course, undefined), or by multiplying both sides by 0 (which will produce equality even when things were originally unequal). For example, multiplying both sides of an equation by x-1 is appropriate only when x ≠ 1.

Algebra

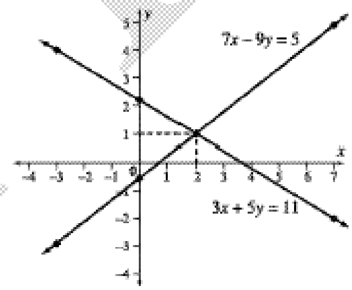
8.8 Understand and solve problems involving linear equations in one and two variables.

8.8a Understand why the solution to the linear equation f(x) = g(x) is the x-value of the intersection of the graphs of f(x) and g(x).



Example: To solve the linear equation 3*x*+1 = *x*+5, graph *f(x)* = 3*x* + 1 and *g(x)* = *x* + 5. The intersection of the graphs of *f*(*x*) and *g*(*x*) shows that the solution to the linear equation is *x* = 2.
Note: Alternatively, the linear equation 3*x* + 1 = *x* + 5 can be rewritten as 2*x* –4 = 0. This yields a single linear function *h*(*x*) = 2*x* –4 which can be solved by graphing h(x) (see end of Grade7).

8.8b Solve simultaneous linear equations in two variables by graphing.



- A linear equation *in two variables* is an expression of the form *ax* + *by* = *c*. The *graph* of such an equation consists of all points (x,y) in the coordinate plane that satisfy the equation.
  Note: Graphs of linear equation in two variables are straight lines.
- A system of *simultaneous linear equations in two variables* consists of two different linear equations in two variables. A *solution* to such a system consists of specific values x_0, y_0 that make both equations true.
- The coordinates of the point of intersection of the graphs of two linear equations are the solutions to the system of equations.
  Example: To solve the system 3*x* + 5*y* = 11; 7*x* - 9*y* = 5, first graph each of the two equations (see figure on the right). The point x_0 = 2 and y_0 = 1 where the graphs of the two equations intersect lies on both graphs, so it is a solution of both equations.

8.8c Solve simultaneous linear equations in two variables by substitution.

8.9 Understand and solve problems involving linear inequalities in one and two variables.

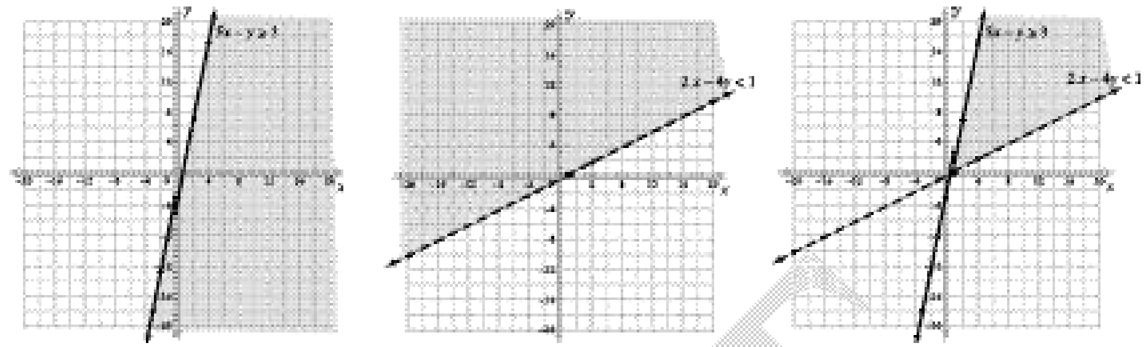
8.9a Understand why the graph of a linear inequality is a half plane.

- In analogy with the vocabulary of equations, the collection of all points (x, y) that satisfy the *linear inequality ax + by ≤ c* is called the *graph* of the inequality. These points lie entirely on one side of the line that is the graph of the equation *ax + by = c.*

8.9b Understand that when both sides of an inequality are multiplied or divided by a negative number the inequality is reversed, but that all other basic operations when applied to both sides preserve the inequality.

8.9c Solve linear inequalities in one and two variables.

Examples*:* Graphs (a) and (b) illustrate that the graph of a linear inequality is a half plane. Graph (c) illustrates a solution to the question: What is the set of points (x, y) that satisfies both 5x –y ≥ 3 and 2x –4y < 1?



(a) (b) (c)

8.10 Know, understand, and use common nonlinear functions, identities, and equations.

8.10a Make regular fluent use of basic algebraic identities.

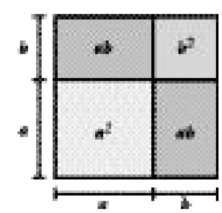
- These include:

  $(a + b)^2 = a^2 + 2ab + b^2$
  $(a - b)^2 = a^2 - 2ab + b^2$
  $(a + b)(a - b) = a^2 - b^2$

  Example: 37 x 43 = (40-3)(40+3) = 1400-9 = 1591.

- Use the distributive law to derive each of these formulas.

  Note: These identities are used in algebraic proofs of the Pythagorean theorem.



- Understand how the partitioned square on the right provides a geometric representation of $(a + b)^2 = a^2 + 2ab + b^2$.

8.10b Recognize common nonlinear functions and their graphs.

- These include:
  -- inverse functions such as $f(x) = k/(bx)$;
  -- quadratic functions such as $f(x) = ax^2 + b$ or $g(x) = (x-a)(x+b)$;
  -- cubic functions, such as $f(x) = x^3$ and $f(x) = x^3 - a$;
  -- square root functions, such as $f(x) = 5\sqrt{x}$;
  -- exponential functions with an integer base, such as $f(n) = 2^n$ for positive integers n.
- Decide if a given graph or table of values suggests an inverse, quadratic, cubic, square root, or exponential function.

8.10c Work fluently with nonlinear functions in geometric contexts of length, area, and volume.

- Examples include:
  -- the area and radius of a circle;
  -- the volume and radius of a sphere;
  -- the number of diagonals and the number of sides of a polygon;
  -- the areas of simple plane figures and their linear dimensions;
  -- the surface areas and volumes of simple three-dimensional solids and their linear dimensions.

8.10d Graph quadratic functions and use the graph to locate roots.

- A *root* of a function f(x) is a value of x for which f(x) = 0.

Note: Beware the confusion inherent in two apparently different meanings of root: the *root of a function* (e.g., $f(x) = 3x^2 -4x + 1$) and the *root of a number* (e.g., $\sqrt{5}$). Although different, these uses do arise from a common source: the root of a number such as $\sqrt{5}$ is the root of an associated equation $f(x) = x^2 - 5$.

- Make tables of values of quadratic functions and plot points by hand.

  Note: To gain experience, students need to do some graphing by hand rather than by using a graphing calculator. However, a calculator can be helpful to fill out a table of values for more complicated functions.

- For quadratic functions that are given in factored form $f(x) = (x - a)(x - b)$, use the roots of the equation $f(x) = 0$ to sketch the graph of $f(x)$.

  Note: This strategy also works for simple quadratic functions that can be factored using an algebraic identity, e.g., $f(x) = x^2 - a^2 = (x+a)(x-a)$.

- Estimate the roots of a quadratic equation from the graph of the corresponding function.

8.10e Solve factorable quadratic equations.

- Include equations of the form $(x+a)^2 = b$.

  Note: In Grade 8, problems involving quadratics should be limited to recognizably factorable examples. General methods of solution such as completing the square, the quadratic formula, and calculator approximations should be deferred until a later grade.