Defining Postsecondary Expectations for Mathematics in Massachusetts

EXECUTIVE SUMMARY

In order to determine what English language arts and mathematics knowledge and skills high school graduates need to enter credit-bearing courses at state colleges and universities, ADP asked postsecondary faculty members in the five partner states (Indiana, Kentucky, Massachusetts, Nevada, and Texas) to:

1. define their expectations of high school graduates (related to admissions and placement decisions);
2. examine what current state standards, high school exit and college entrance and placement assessments currently expect of students; and
3. identify the gaps that may exist among these various sets of expectations.

ADP's gap-analysis work consisted of three parts.[1] In **part one,** English Language Arts (ELA) and mathematics faculty members from K-12 systems and from two- and four-year colleges in the five ADP partner states examined the content of partner-state high school graduation assessments, national college admissions and placement tests, a sampling of postsecondary institutional placement tests, and the GED. The results from part one comprise Education Trust's five state reports, which reflect the feedback received from faculty members to the aforementioned assessments. These reports discuss the relative strengths and weaknesses of the assessments and how well each assessment might serve postsecondary institutions in making admissions and placement decisions. By examining individual test items, the faculty members were able to examine the breadth and depth of content coverage, as well as the types and quality of test items.[2]

In **part two**, using its assessment-to-standards alignment protocol, Achieve conducted studies of the ADP partner states' ELA and mathematics graduation assessments and the corresponding sets of state standards. The results from part two comprise Achieve's ten state alignment reports (i.e., five ELA, five mathematics). These reports discuss

[1] The **Education Trust** gap-analysis reports were originally circulated in 2002 and key findings have been incorporated into this report. The summary tables from the **Achieve** alignment study can also be found in the appendices of this report, and the findings from the **ADP** postsecondary discussions are presented here in this report for the first time. For more information, please visit the **ADP** website <http://www.americandiplomaproject.org/>.

[2] The Education Trust also prepared an analytical "cross-state" report that summarizes their findings.

how closely the expectations articulated in the state standards are measured by the state's high school assessments. Achieve matched each individual test item to the standards, and analyzed how well the items measure students' mastery of the knowledge and skills described in those standards. Achieve was able to report how effectively the assessments measure the states' expectations for high school graduation.

In **part three**, ADP staff met with faculty members from two- and four-year colleges in each of the ADP partner states, representing a broad range of content areas (organized into the "ELA" and "Math" Teams) to:

1. define their expectations for incoming students,
2. prioritize their expectations for students, as may be contained in the state's academic standards;
3. determine the degree to which the current standards and assessments together reflect those expectations, and
4. identify any gaps (missing content) in the standards and assessments.

In this round of discussion, it was particularly helpful to have had a cross-section of faculty from the humanities, sciences and social sciences, since reading, writing and mathematics skills are necessary for success in all credit-bearing courses throughout college, not just in English and mathematics courses.

Following are the essential findings from the "gap analysis":

1. The Massachusetts standards for mathematics contained in the Curriculum Frameworks reflect postsecondary expectations for incoming students to do credit-bearing coursework without the need for remediation.

2. The Grade 10 Mathematics MCAS is a strong test, but it would need some adjustments before being wholly useful to postsecondary institutions for the purposes of admission or placement. The lack of Algebra 2 items on MCAS was the central content gap. (See discussion of findings below for additional specific suggestions for improvement.)

3. A predicative validity study needs to be conducted to see how closely performance on the Grade 10 Mathematics MCAS predicts postsecondary success and how the MCAS scores correlate to the SAT 1 Mathematics, Accuplacer and other national tests that postsecondary institutions currently used for admissions and placement.

4. Grade 10 Mathematics MCAS results ought to be reported on high school transcripts. To maximize the usefulness of these results, they ought to be broken down by domain or strand.

A detailed discussion of the findings follows.

DISCUSSION OF FINDINGS

In Massachusetts, through the three-part process described above, faculty identified how closely the Mathematics Curriculum Frameworks and the Grade 10 Mathematics MCAS assessment aligned to postsecondary expectations. The following discussion of findings has been organized by the fundamental issues/questions that arose during this review process.

Issues/Questions:

1. What are the postsecondary expectations for mathematics in Massachusetts?

2. How closely do the Massachusetts Mathematics Curriculum Frameworks align to the postsecondary expectations for students in mathematics?

3. How closely does the Grade 10 Mathematics MCAS align to the expectations articulated in the Massachusetts Mathematics Curriculum Frameworks?

4. How closely do the emphases on the Grade 10 Mathematics MCAS align to the emphasis that postsecondary faculty place on the mathematics knowledge and skills needed to begin college-level work?

5. Is the Grade 10 Mathematics MCAS useful – as is – to postsecondary institutions for the purposes of admission or placement?

6. What changes or adjustments to the Grade 10 Mathematics MCAS do postsecondary faculty recommend in order to make that assessment more useful to postsecondary institutions for the purposes of admission or placement?

#1 Issue/Question: What are the postsecondary expectations for mathematics in Massachusetts?

Answer: Members of the Massachusetts postsecondary community linked basic college-readiness to the completion of the three courses in the traditional sequence of Algebra 1, Geometry, and Algebra 2 (ideally followed by a fourth year of mathematics).

The following table lists the specific mathematics knowledge and skills they identified as necessary for students entering credit-bearing college mathematics courses, that is without the need for remediation (see Math Table 1).

Math Table 1: Minimal Math Skills Needed to Avoid Remedial Math

Number Sense

1. Basic arithmetic skills with fluency & without calculator
2. Relationships among fractions, decimals, and percentages (representations of parts of numbers)
3. Ratio and proportion
4. Estimation and reasonableness of answers
5. Ability to read and comprehend math problems (word/verbal problems)
6. Unit conversion and measurement (metric, especially for science and health fields)
7. Basic rules of exponents, operations with square roots, scientific notation

Basic Geometric Properties and Concepts

8. Basic characteristics of parallelograms, triangles, circles
9. Area, perimeter, volume for plane or solid quadrilaterals (recognize, understand and apply formulas)
10. Concepts of perpendicularity, parallelism, similarity
11. Pythagorean Theorem
12. Properties and measurement of angles
13. Proofs and logic, reasoning and deduction skills

Algebra

14. Concept of variable and basic symbolic manipulation
15. Solve (assuming simplification) linear equations and inequalities algebraically and graphically (including literal) [note: students may not retain this level of information]
    a. Slope
    b. Intercepts
    c. Writing equation/inequality from a graph
    d. Slope, intercepts, and 2-point formulas
16. Solve 2x2 systems of linear equations
17. Solve basic quadratic equations by factoring and the quadratic formula
18. Understands basic graphing techniques
    a. Linear
    b. Quadratic
19. General notation of a function; evaluation of linear and non-linear functions
20. Operations on polynomials, including rationals
21. Simplify algebraic expressions, including elementary rational expressions

Probability and Statistics

22. Interpret and make simple charts, tables, graphs)
    a. Line
    b. Bar
    c. Circle
23. Mean, median, mode, range, outliers, frequency
24. Sampling methods

In addition, postsecondary faculty identified supplementary knowledge and skills that students need in order to excel in those credit-bearing courses (see Math Table 2).

Math Table 2: Math Skills Needed for a High Degree of Success in Credit-Bearing Courses

25. Ability to apply, integrate, and connect the knowledge and skills listed above
26. Deeper understanding of math concepts and skills and more confidence in math skills
27. Formal and informal algebraic and geometric proofs and logical reasoning and deductive skills
28. Familiarity with exponential functions

Finally, postsecondary faculty identified three other pieces of mathematics knowledge that incoming students, who aspire to enter mathematics (or math-dependent) majors, would need:

a. Exposure to complex numbers
b. Familiarity with trigonometric functions
c. Familiarity with logarithmic functions

#2 Issue/Question: How closely do the Massachusetts Mathematics Curriculum Frameworks align to the postsecondary expectations for students in mathematics?

Answer: In general, according to postsecondary faculty who reviewed the standards, the Grade 10 Mathematics Standards are strong; students who master their content will be well prepared for credit-bearing postsecondary mathematics courses. In some areas, math and science professors felt that what the state is expecting is excessive.

ADP asked members of the Massachusetts postsecondary community individually to prioritize, from their perspective, the content/competencies described in each of the thirty Grade 10 Mathematics Standards on a scale from "0" to "3" ("0" meaning "extraneous," "1" meaning nice, but "can-live-without," "2" meaning "important," and "3" meaning "vital"). According to their collective judgment, half of the 30 Grade 10 standards "important" or "vital."

Math Table 3: Mathematics Standards Priority v. the Grade 10 MCAS

Average Priority Assigned to a Standard	Number of Standards per Priority Level	Number of Test Items Aligned to These Standards
3.0	2	3
2.5-2.9	4	2
2.0-2.4	9	9
1.5-1.9	7	6
1.0-1.4	7	4

Average Priority Assigned to a Standard	Number of Standards per Priority Level	Number of Test Items Aligned to These Standards
0.0-0.9	1	1
TOTAL	30	25

Not only do the standards reflect the vital content identified by the faculty members, but the grade 10 MCAS test favors that vital content: more than half of the 25 grade 10 MCAS test items are aligned to the important or vital standards (see Math Table 3).

Two important issues arose during the prioritization exercise. First, faculty found the task of prioritizing the standards complicated by the number of different expectations contained within an individual standard (i.e., some parts were deemed "vital," other parts not). Second, some significant differential existed between science team members (physics and biology) and math (and economics) team members. Much more content included in the standards was deemed crucial by science faculty. In fact, the non-mathematics faculty members were five times more likely to grant importance to a standard than were the mathematics faculty. The following are examples of standards that were given a more significant priority by the science faculty:

- STANDARD 10.G.2: Draw congruent and similar figures using a compass, straightedge, protractor and other tools such as computer software. Make conjectures about methods of construction. Justify the conjectures by logical arguments.

- STANDARD 10.M.3: Relate changes in the measurement of one attribute of an object to changes in other attributes, e.g., how changing the radius or height of a cylinder affects its surface area or volume.

- STANDARD 10.D.1: Select, create, and interpret an appropriate graphical representation (e.g., scatterplot, table, stem-and-leaf plots, box-and-whisker plots, circle graph, line graph, and line plot) for a set of data and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.

- STANDARD 10.D.3: Describe and explain how the relative sizes of a sample and the population affect the validity of predictions from a set of data.

Measurement, graphical displays, and interpretation of data are all critical aspects of scientific study.

There were also two criticisms of the standards. One criticism was that while the introductory philosophy statement includes expectations for rich processes (e.g., reasoning and communication), these processes are not clearly described in the standards themselves. A second criticism was raised about the core content standards versus the state's course level descriptions. Content in the course level descriptors does not always appear in the grade 9-12 core content standards. For example, formal proofs and logical reasoning – content faculty members think is important – are missing from the core content, but are included in the course descriptors.

#3 Issue/Question: How closely does the Grade 10 Mathematics MCAS align to the expectations articulated in the Massachusetts Mathematics Curriculum Framework?

Answer: Results of the Achieve assessment-standards alignment study, determined that the Grade 10 Mathematics MCAS aligns closely to the Massachusetts Mathematics Curriculum Framework for Grade 10. Postsecondary faculty felt, however, that the standards are written at a much higher level of complexity and greater depth than are many of the items on the MCAS.

The assessment-standards alignment study (using the Achieve item map[3]) revealed that the content of a majority of the test items on the Grade 10 Mathematics MCAS are clearly consistent with the content described in the standards (51.2 percent). Another 41.5 percent of the test items received a "1b"[4], meaning that they were aligned to only part of the content in a standard, often to the less demanding part. Given that many standards contain several expectations, reviewers were no very concerned about this fact. The remaining three test items were found to be inconsistent (see Math Table 4).

In addition (Achieve map), more than a majority of test items received "2s" for clearly consistent performance centrality (56.1 percent). A score of "2" means that there is a strong match between the type of performance presented by each test item and the type of performance described by the corresponding standard. Another 39.0 percent received a "1b," meaning that the test items measured only a portion of the – rather than the entire -- performance described in the standards. Often, as stated above, many Massachusetts' standards include several performances in one. For example, they include "identify" and "use" or "extend," "analyze," and

[3]Massachusetts made available the "item map" for the Grade 10 Mathematics MCAS. The map describes which items are meant to assess which standards. After reviewing the state map, Achieve reassigned six of the MCAS items to different stands and/or standards. The data provided below and in the appendices include two alignment analyses reflecting the different item maps.

[4] For a complete description of the Achieve alignment ratings and categories, please see appendix 1 below.

"describe." Once again, a small number of test items (two) received a "0", indicating that the performance demanded on the assessment failed to match the performance demanded in the standards (see Math Table 4).

Math Table 4: Content & Performance Centrality of Grade 10 Mathematics MCAS

Centrality	# of Items	2 (clearly consistent)	1a (standard not specific enough)	1b (item assesses only a part of the standard)	0 (inconsistent)
Content (*State Map*)	41 100%	17 41.5%	0	18 43.9%	6 14.6%
Content (*Achieve Map*)	41 100%	21 51.2%	0	17 41.5%	3 7.3%
Performance (*State Map*)	41 100%	20 48.8%	1 2.4%	16 39.0%	4 9.8%
Performance (*Achieve Map*)	41 100%	23 56.1%	0	16 39.0%	2 4.9%

Again, the inclusion of more than one (but related) knowledge and/or skill within many of the standards is the reason for many of the content and performance centrality "1b"s. This does not reflect a weak alignment of the MCAS to the Curriculum Framework, but is simply a reflection of the way the individual standards have been articulated.

Under all five strands of the Curriculum Framework, there is at least one Grade 10 Mathematics MCAS item that aligns to a Grade 8 standard. For the most part, this is not problematic because the majority of test items are written at the high school level. The only exception is in the case of the "Data Analysis" strand, under which seven of the eight items on the test aligned to a Grade 8 rather than a Grade 10 standard.

With regard to the level of cognitive demand on the Grade 10 Mathematics MCAS (Achieve map), as the following table demonstrates, 85.4 percent of the test items were rated a level "2" or below; close to ten percent were not rated because they contain an inappropriate source of challenge. Roughly five percent of the test items rated a "3," requiring "strategic thinking" (see Math Table 5). The state map includes almost identical percentages.

Math Table 5: Level of Cognitive Demand of Grade 10 Mathematics MCAS

Map	# of Items	4 (extended thinking)	3 (strategic thinking)	2 (skill/ concept)	1 (recall)	Not Scored
State Map	41 100%	0	2 4.9%	20 48.8%	14 34.1%	5 12.2%
Achieve Map	41 100%	0	2 4.9%	20 48.8%	15 36.6%	4 9.7%

Cross-content postsecondary faculty identified the set of Algebra items to be the most rigorous and to include a good mix of items. Three of the 10 test items are open-response and two are short-answer, with the remaining five being multiple choice. This mix of test item formats contributes to an enhanced level of challenge for this item set. Faculty identified the set of test items in Number Sense to be the least challenging.

K-12 and postsecondary ELA and mathematics faculty found the demands of the MCAS open-response items to be a distinguishing characteristic of the assessment. The test items, they noted, required higher orders of thinking such as creating graphs, explaining steps taken and justifying answers given.

Reviewers expressed concern, however, that the MCAS retest administered in the Fall of 2002 was pitched at a lower level of cognitive challenge.[5] They found significantly more middle-school level items (36 percent up from 20 percent on the regular test). They also stated that the open-response items were not as challenging.

#4 Issue/Question: How closely do the emphases on the Grade 10 Mathematics MCAS align to the emphases that postsecondary faculty places on the mathematics knowledge and skills needed to begin college-level work?

Answer: The fact that MCAS does not include any Algebra 2 items is problematic in terms of using the test for college entrance or placement purposes. Postsecondary faculty members would place more emphasis on algebra and number sense than the MCAS currently does.

If charged with developing the high school assessment, postsecondary faculty would divide nearly 80 percent of the MCAS between "Number Sense and Operations" and "Patterns, Relations, and Algebra" strands. They would distribute the remaining 20 percent among the remaining three strands. The Grade 10 Mathematics MCAS items, whether mapped by the state or by Achieve, are currently divided much more evenly across

[5] The MCAS retests are pitched at a lower level of cognitive challenge because they are not designed to measure and report student performance at the *Proficient or Advanced levels*. For more information, visit the MCAS Retest website at http://www.doe.mass.edu/mcas/retest.html.

all strands. Consequently, much less emphasis is placed on "Number Sense and Operations" and less emphasis on "Patterns, Relations, and Algebra," and more emphasis on the remaining three Strands (see Math Table 6).

Math Table 6: Mathematics Standards Emphasis/Balance on the Grade 10 MCAS

GRADE 10 MATH CURRICULUM FRAMEWORK STRANDS	Desired emphasis (postsecondary faculty)	Current GRADE 10 emphasis (STATE MAP)	GRADE 10 emphasis (ACHIEVE MAP)
NUMBER SENSE AND OPERATIONS	39.2%	22.0%	14.6%
PATTERNS, RELATIONS, AND ALGEBRA	40.0%	24.4%	34.1%
GEOMETRY	8.3%	24.4%	22.0%
MEASUREMENT	5.8%	9.8%	9.8%
DATA ANALYSIS, STATISTICS, AND PROBABILITY	6.7%	19.5%	19.5%
TOTAL	100.0%	100.0%	100.0%

Postsecondary faculty expressed surprise by the relatively little emphasis placed on the "Number Sense and Operations" Strand on the Grade 10 Mathematics MCAS because so many incoming college students are weak in that area. While the professors' first choice would be to increase the number of test items dedicated to Number Sense and Operations, a smaller number of items on the MCAS would be acceptable, as long as the focus of these test items were shifted to add:

- more complex, "problem-solving" questions, and
- more test items dealing with rational numbers, percents, decimals, and square roots.

Postsecondary faculty also expressed surprise that so much relative emphasis was placed on the "Data Analysis, Statistics, and Probability" Strand on the Grade 10 Mathematics MCAS because most postsecondary departments provide the necessary statistical training to their students. If incoming high school graduates have mastered basic graphing and interpretation skills, faculty asserted, then they will be adequately prepared to begin credit-bearing college coursework.

The emphasis on Geometry on the MCAS, though greater than the postsecondary faculty member would give, was acceptable, but more test

items dealing with geometric proof would be an improvement from the postsecondary perspective. One way to accomplish this would be to include items on the assessment that would provide students with a list of possible rationales from which the students would be required to select the correct one.

#5 Issue/Question: Is the Grade 10 Mathematics MCAS useful – as is – to postsecondary institutions for the purposes of admission or placement?

Answer: Both K-12 and postsecondary faculty found that while the MCAS does not indicate college readiness in mathematics, neither does the SAT 1. Neither test includes enough Algebra 2 content. In addition, both the MCAS and the SAT 1 include too much mathematics linked to the Grade 8 or middle school content. The Accuplacer, a computer-administered test published by the College Board that is used nationally – and in Massachusetts – for college placement did not fare any better. The vast majority of the items on the Accuplacer elementary algebra test address Algebra 1 content; not even ten percent of the test items address Algebra 2.

The MCAS is first administered in the Spring of Grade 10. As such, vital Algebra 2 content (not yet encountered by the end of sophomore year) is not assessed on the MCAS. Faculty expressed a preference for a graduation test administered in Grade 11, as it could include more vital content and would be administered closer in time to college entrance. It would thereby increase the potential usefulness of the test data to colleges and universities.

Beyond the timing of the MCAS, postsecondary faculty also expressed concern about the fact that the "pass" score on the MCAS corresponds to a "needs improvement" threshold of performance rather than to "proficient" performance. In fact, students taking the Grade 10 Mathematics MCAS could answer all of the Grade 8 questions correctly and be more than three quarters of the way to a passing score, needing to answer only one in five of the remaining 25 questions correctly. Not surprisingly, then, a passing score on the Grade 10 Mathematics MCAS may indicate that a student is "on the path" to college readiness if one assumes that test-takers will take additional mathematics courses in Grades 11 and 12, but it is not definitive. As one faculty member put it, "the current graduation exam is a necessary but not a sufficient indication of readiness for credit bearing courses in college." On the other hand, a failing score on MCAS would indicate that a student would need significant intervention in order to be college-ready by the end of high school.

Faculty members also identified several strengths of the current MCAS assessment, including

- the restricted use of calculators (calculators are allowed on two of three sections of the test);

- the sophistication of many of the assessment questions that require students to reason;
- many problems on the test ask students to use knowledge of two or more domains/strands in order to solve them; and
- a good balance of short answer, open response and multiple choice questions.

#6 Issue/Question: What changes to the Grade 10 Mathematics MCAS do postsecondary faculty recommend in order to make the assessment more useful to postsecondary institutions for the purposes of admission or placement?

Answer: Faculty had several recommendations to make the mathematics assessments more useful to postsecondary institutions.

First, while postsecondary faculty members are familiar with what student performance on both the SAT 1 Mathematics and the Accuplacer means for placement purposes, they were not familiar with the meaning of student performance on the Grade 10 Mathematics MCAS. They recommended that Massachusetts conduct a predicative validity study to see how closely performance on the Grade 10 Mathematics MCAS predicts postsecondary success. They also recommended that Grade 10 Mathematics MCAS scores be correlated with the SAT 1 Mathematics and other national tests.

Second, postsecondary faculty members recommended that student high school transcripts include Grade 10 Mathematics MCAS scores reported by strand or domain (e.g., Algebra, Geometry, Number Sense, and Data Analysis).

Third, as noted earlier, the current cut score for high school graduation of 220 indicating "needs improvement" is not high enough to indicate college readiness. Postsecondary faculty members support the raising of the passing score to proficient. With the graduation requirement set at "needs improvement," high school students could be misled into thinking that the math needed to pass the Grade 10 Mathematics MCAS is equivalent to being college ready.

Fourth, postsecondary faculty members expressed the need for students to understand that they must continue to increase their mathematics knowledge and skills throughout their years in high school, certainly beyond what is needed to pass the MCAS, in order to be ready for credit-bearing postsecondary mathematics courses. If local districts don't require students to take higher-level math courses, they asserted, – and four years of math -- students won't choose to do it on their own.

APPENDICES

APPENDIX 1: METHODS

What follows is further detail about the methods used in the three parts of The ADP's gap-analysis work:

- In part one, the Education Trust assembled faculty members from K-12 systems and from two- and four-year colleges to examine the content of partner-state high school graduation assessments, national college admissions and placement tests;

- In part two, Achieve staff, using an assessment-to-standards alignment protocol, conducted studies of the ADP partner states' English Language Arts and mathematics graduation assessments and the corresponding sets of state standards; and

- In part three, ADP staff met with faculty members from two- and four-year colleges in each of the ADP partner states to focus on defining postsecondary expectations and to determine the degree to which current ADP state standards and assessments reflect those expectations.

ED TRUST PROTOCOL

The Education Trust mathematics protocol organizes information in two dimensions:

- **Content:** what knowledge, concepts, and skills are being assessed? In mathematics, content included course title/level, content/topic and state standard.

- **Demand:** how demanding are the test items? In mathematics, teams evaluated context, intricacy and kind of knowledge.

  How did reviewers describe demand in mathematics? The mathematics reviewers had two categories that helped them to systematically describe the kind of demands of particular tests: context and kind of knowledge, as follows:

  - **CONTEXT:** Simply, is there an attempt at a real-world situation, or not? This category is answered either yes or no. Context is described as a difference in the way the problem is presented. No context items lack a verbal or situational context and are usually presented in numbers, often with mathematical symbols.

  - **KIND OF KNOWLEDGE TESTED:** What is the characteristic, or "kind," of knowledge is tested? The protocol adapted the three "kinds" of knowledge identified by NAEP: procedural, conceptual understanding and problem solving. It added one for recall/recognition.

- **Recall/recognition** refers to recalling or recognizing mathematical concepts, facts, terminology and principles. Recall/Recognition items generally lack depth, but instead ask test-takers to show superficial understanding.

- **Procedural** means the correct response can be produced by applying a practiced algorithm or process when the necessary information is given. Procedural items ask test-takers to "do math" as opposed to conceptual items which ask for "understanding math".

- **Conceptual** understanding refers to understanding mathematical concepts, ideas, constructs or theories. Some explanation, modeling or representation may be called for.

- **Problem solving** items require test-takers to use both conceptual and procedural knowledge. They likely require some interpretation, as well, and the approach may not be immediately obvious. In addition, there may be some missing or hidden information that test-takers must deduce. Problem solving items also typically ask for explanation.

ACHIEVE ASSESSMENT ALIGNMENT PROTOCOL

The Achieve mathematics protocol considers four dimensions in its analysis of the degree of alignment between an assessment and a set of standards.

- **Content centrality:** This criterion provides a deeper analysis of the match between the content of each examination question and the content of the related standard by examining the degree or quality of the match. Reviewers assign each item to one of four categories based on the degree of alignment: "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

- **Performance centrality:** This criterion focuses on the degree of the match between the type of performance (cognitive demand) presented by each examination item and the type of performance described by the related standard. Each item makes a certain type of cognitive demand on a student (e.g., the item requires a certain performance such as "select," "identify," "compare," or "analyze"). Reviewers assign each item to one of four categories based on the degree of alignment: "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

- **Challenge:** This criterion is applied to a set of items to determine whether doing well on these items requires students to master challenging subject matter. Reviewers consider two factors in evaluating sets of examination items against the challenge criterion: source of challenge and level of challenge.

    - **Source of challenge** attempts to uncover whether the individual examination items in a set are difficult because of the knowledge and skills they target, or

because of other reasons not related to the subject matter, such as relying unfairly on students' background knowledge. Reviewers rate each item as having an appropriate (1) or inappropriate (0) source of challenge.

- **Level of challenge** compares the emphasis of performance required by a set of items to the emphasis of performance described by the related standard. In addition to evaluating alignment, reviewers also judge whether the set of examination items has a span of difficulty appropriate for students at a given grade level based on the standards, the assessment and supporting materials. Reviewers assign each item to one of four categories indicating its type of cognitive demand: 4 = extended thinking; 3 = strategic thinking; 2 = skill/concept; 1 = recall.

- **Balance and range:** No one assessment can measure the full range of knowledge and skills described in the state standards. Evaluating balance and range provides both qualitative and quantitative descriptive information about the choices states or test developers have made.

  - **Balance** compares the emphasis of content supplied by an item set to the emphasis of content described by the standards. In addition to evaluating alignment, reviewers also judge whether the set of items emphasizes the more important content at the grade level. Reviewers write a succinct summary of the balance of each item set.

  - **Range** is a measure of coverage or breadth (the numerical proportion of all content addressed).

ADP PROTOCOL

The discussions at the ADP meetings were structured according to a set of guiding questions (and tasks) that the participants were given in advance of the meeting.

A. Define Postsecondary Content and Competencies:

1. Briefly outline the math content and competencies that students must know and be able to do if they are NOT to be placed into remedial math courses.

2. In addition, briefly outline what math content and competencies you would add to the core list above to prepare students TO DO WELL in credit-bearing freshman courses and to be prepared to do well in upper-level courses in your field.

B. Evaluate the state high school mathematics standards:

With your core competencies in mind, we would like you to evaluate the state's mathematics standards. Within the limitations of administering large-scale, on-demand assessments, please identify the emphasis (in terms of a percentage from one to one hundred) that the different standards should be given on a state test.

First, within the limitations of administering large-scale, on-demand assessments, please identify the emphasis (in terms of a percentage from one to one hundred) that different Strands should be given on a state test.

Second, with respect to the Standards listed under each Strand, please assign a priority to the content/competency, given what you would expect from freshmen students. Here is the rubric:

- Give a "3" for vital content.
- Give a "2" for important content.
- Give a "1" for nice, but "can-live-without" content.
- Give a "0" for extraneous, "Who cares?" content.

You can have as many "3s", "2s", etc. as you would like. In the example below, a "2" indicates that the reviewer believes it is important (though not vital) for students entering college to be able to "locate, evaluate, and apply information for a realistic purpose."

C. Summary questions about the state's high school mathematics standards:

These are questions for your consideration; you will discuss them in breakout sessions at the meeting.

1. If students master the state's high school mathematics standards, will they have the competencies they need
   - to avoid placement in remedial college mathematics classes? If not, why not?
   - to do well in freshman credit-bearing mathematics courses? If not, why not?
   - to do well in your course (including non-math courses such as chemistry or economics)? If not, why not?

2. Are there any serious gaps in content or competencies outlined in the high school mathematics standards? Think back to your list of core competencies; are all of them represented? If you identify gaps, please note them.

3. Would a student who meets the expectations of the high school mathematics standards sufficiently meet your expectations for your incoming students? Why or why not?

4. Do the high school mathematics standards show an appropriate balance of conceptual, procedural and practical problem-solving?

5. Is there an appropriate range of math content required of students?

6. What are the key points that you would like to see reported to state about the high school mathematics standards?

APPENDIX 2: EDUCATION TRUST RESULTS

The following table represents an "At-a-Glance" summary of results from the Education Trust gap analysis of the Massachusetts Comprehensive Assessment System (MCAS): Grade 10 Mathematics and the SAT 1 MATHEMATICS.

COURSE/GRADE LEVEL	TESTS (% of total items):	
	Grade 10 Math MCAS	SAT 1 Math
Elementary	0%	0%
Middle School	24%	40%
Algebra 1	52%	32%
Geometry	24%	18%
Algebra 2	0%	8%
Other	0%	2%

KIND OF KNOWLEDGE	TESTS (% of total items):	
	Grade 10 Math MCAS	SAT 1 Math
Procedure	51%	75%
Concept	32%	7%
Problem Solving	13%	15%
Recall	4%	3%
Other	0%	0%

CONTENT DISTRIBUTION	TESTS (% of total items):	
	Grade 10 Math MCAS	SAT 1 Math
Number	5.7%	25%
Measure	13%	5%
Geometry	15%	18%
Algebra	37.7%	37%
Calculus	0%	0%
Statistics	26%	10%
Discrete	1.9%	3%
Reason	0%	2%
Other	0%	0%

APPENDIX 3: ACHIEVE ASSESSMENT-TO-STANDARDS ALIGNMENT STUDY

The following tables represent the results from the Achieve alignment study for the Massachusetts Comprehensive Assessment System (MCAS) in Mathematics and the Massachusetts Mathematics Curriculum Framework.

Assessment
Massachusetts Comprehensive Assessment System (MCAS):
Grade 10 Mathematics, Form 1, Spring 2002

Standards
Massachusetts Mathematics Curriculum Framework
Adopted November 2000

Test Design
31Multiple-Choice Items
6 Open-Response Items
4 Short-Answer Items
41 Total number of test items

Item Coverage

Massachusetts Curriculum Mathematics Framework	State Map		Achieve Map	
	# of Items	% of Test	# of Items	% of Test
Number Sense, and Operations	9	22.0%	6	14.6%
• Standard 10.N.1	2		2	
• Standard 10.N.2	3		1	
• Standard 10.N.3	1		1	
• Standard 10.N.4	2		1	
• Standard 8.N.10	1		1	
Patterns, Relations, and Algebra	10	24.4%	14	34.1%
• Standard 10.P.1	1		1	
• Standard 10.P.2	0		0	
• Standard 10.P.3	0		0	
• Standard 10.P.4	2		2	
• Standard 10.P.5	0		1	
• Standard 10.P.6	0		1	
• Standard 10.P.7	2		2	
• Standard 10.P.8	2		3	
• Standard 8.P.2	1		1	
• Standard 8.P.3	0		1	
• Standard 8.P.5	1		1	
• Standard 8.P.7	1		1	
Geometry	10	24.4%	9	22.0%
• Standard 10.G.1	0		0	

Massachusetts Curriculum Mathematics Framework	State Map		Achieve Map	
	# of Items	% of Test	# of Items	% of Test
• Standard 10.G.2	0		0	
• Standard 10.G.3	0		0	
• Standard 10.G.4	0		0	
• Standard 10.G.5	2		2	
• Standard 10.G.6	1		1	
• Standard 10.G.7	1		0	
• Standard 10.G.8	0		0	
• Standard 10.G.9	2		2	
• Standard 10.G.10	0		0	
• Standard 10.G.11	1		1	
• Standard 8.G.2	2		2	
• Standard 8.G.3	1		1	
Measurement	4	9.8%	4	9.8%
• Standard 10.M.1	4		2	
• Standard 10.M.2	0		1	
• Standard 10.M.3	0		0	
• Standard 10.M.4	0		0	
• Standard 8.M.3	0		1	
Data Analysis, Statistics, & Probability	8	19.5%	8	19.5%
• Standard 10.D.1	1		1	
• Standard 10.D.2	0		0	
• Standard 10.D.3	0		0	
• Standard 8.D.2	2		2	
• Standard 8.D.3	2		2	
• Standard 8.D.4	3		3	
TEST AS A WHOLE	41	100.0%	41	100.0%

Content Centrality (State Map)

Note: Ratings for Content Centrality are "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

Standard	# of Items	2	1a	1b	0
Number Sense & Operations	9 22%	2	0	3	4
Patterns, Relations, & Algebra	10 24.4%	5	0	4	1
Geometry	10 24.4%	7	0	2	1
Measurement	4 9.7%	0	0	4	0
Data Analysis, Statistics, & Probability	8 19.5%	3	0	5	0

Standard	# of Items	2	1a	1b	0
TEST AS A WHOLE	41 100%	17 41.5%	0	18 43.9%	6 14.6%

Content Centrality (Achieve Map)
Note: Ratings for Content Centrality are "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

Standard	# of Items	2	1a	1b	0
Number Sense & Operations	6 14.6%	2	0	3	1
Patterns, Relations, & Algebra	14 34.1%	8	0	5	1
Geometry	9 22%	7	0	1	1
Measurement	4 9.8%	1	0	3	0
Data Analysis, Statistics, & Probability	8 19.5%	3	0	5	0
TEST AS A WHOLE	41 100%	21 51.2%	0	17 41.5%	3 7.3%

Performance Centrality (State Map)
Note: Ratings for Performance Centrality are "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

Standard	# of Items	2	1a	1b	0
Number Sense & Operations	9 22%	3	1	3	2
Patterns, Relations, & Algebra	10 24.4%	4	0	6	0
Geometry	10 24.4%	5	0	3	2
Measurement	4 9.7%	4	0	0	0
Data Analysis, Statistics, & Probability	8 19.5%	4	0	4	0
TEST AS A WHOLE	41 100%	20 48.8%	1 2.4%	16 39.0%	4 9.8%

Performance Centrality (Achieve Map)
Note: Ratings for Performance Centrality are "2" = clearly consistent; "1a" = not specific enough; "1b" = somewhat consistent; "0" = inconsistent.

Standard	# of Items	2	1a	1b	0
Number Sense & Operations	6 14.6%	3	0	2	1
Patterns, Relations, & Algebra	14 34.1%	7	0	7	0
Geometry	9 22%	5	0	3	1
Measurement	4	4	0	0	0

Standard	# of Items	2	1a	1b	0
	9.8%				
Data Analysis, Statistics, & Probability	8 19.5%	4	0	4	0
TEST AS A WHOLE	41 100%	23 56.1%	0	16 39.0%	2 4.9%

Source of Challenge (State Map)
Note: Ratings for Source of Challenge are "1" = appropriate, "0" = inappropriate.

Standard	# of Items	1	0
Number Sense & Operations	9 22%	7	2
Patterns, Relations, & Algebra	10 24.4%	9	1
Geometry	10 24.4%	9	1
Measurement	4 9.7%	4	0
Data Analysis, Statistics, & Probability	8 19.5%	7	1
TEST AS A WHOLE	41 100%	36 87.8%	5 12.2%

Source of Challenge (Achieve Map)
Note: Ratings for Source of Challenge are "1" = appropriate, "0" = inappropriate.

Standard	# of Items	1	0
Number Sense & Operations	6 14.6%	5	1
Patterns, Relations, & Algebra	14 34.1%	13	1
Geometry	9 22%	8	1
Measurement	4 9.8%	4	0
Data Analysis, Statistics, & Probability	8 19.5%	7	1
TEST AS A WHOLE	41 100%	37 90.2%	4 9.8%

Level of Cognitive Demand (State Map)
Note: Ratings for Level of Cognitive Demand are “4” = extended thinking; “3” = strategic thinking; “2” = skill/concept; “1” = recall.

Standard	# of Items	4	3	2	1	Not Scored
Number Sense & Operations	6 14.6%	0	0	1	6	2
Patterns, Relations, & Algebra	14 34.1%	0	1	4	4	1

Standard	# of Items	4	3	2	1	Not Scored
Geometry	9 22%	0	0	7	2	1
Measurement	4 9.8%	0	0	3	1	0
Data Analysis, Statistics, & Probability	8 19.5%	0	1	5	1	1
TEST AS A WHOLE	41 100%	0	2 4.9%	20 48.8%	14 34.1%	5 12.2%

Level of Cognitive Demand (Achieve Map)

Note: Ratings for Level of Cognitive Demand are "4" = extended thinking; "3" = strategic thinking; "2" = skill/concept; "1" = recall.

Standard	# of Items	4	3	2	1	Not Scored
Number Sense & Operations	6 14.6%	0	0	0	5	1
Patterns, Relations, & Algebra	14 34.1%	0	1	6	6	1
Geometry	9 22%	0	0	6	2	1
Measurement	4 9.8%	0	0	3	1	0
Data Analysis, Statistics, & Probability	8 19.5%	0	1	5	1	1
TEST AS A WHOLE	41 100%	0	2 4.9%	20 48.8%	15 36.6%	4 9.7%

Range (State Map)

Note: All grade 8 and grade 12 indicators that are in the five content strands and eligible for state assessment are included in these tallies.

Standard	Portion of Standards Assessed
Number Sense & Operations	3/4 or 0.75
Patterns, Relations, & Algebra	5/8 or 0.63
Geometry	5/10 or 0.50
Measurement	1/3 or 0.33
Data Analysis, Statistics, & Probability	1/3 or 0.33
TEST AS A WHOLE	15/28 or 0.54

Range (Achieve Map)

Note: All grade 8 and grade 12 indicators that are in the five content strands and eligible for state assessment are included in these tallies.

Standard	Portion of Standards Assessed
Number Sense & Operations	3/4 or 0.75
Patterns, Relations, & Algebra	6/8 or 0.75
Geometry	4/10 or 0.40
Measurement	2/3 or 0.67

Standard	Portion of Standards Assessed
Data Analysis, Statistics, & Probability	1/3 or 0.33
TEST AS A WHOLE	16/28 or 0.57

RANGE COMMENTS

1) Four standards are assumed for the Number Sense and Operation strand although the state indicates that standard 10.N.3 would not be assessable in 2001. Item 6 was judged to align with standard 10.N.3, so it is counted in this calculation.
2) Ten, rather than eleven, standards are assumed for the Geometry strand since the state indicates that standard 10.G.8 would not be assessable in 2001. There are no items mapped to this standard.
3) Three, rather than four, standards are assumed for the Measurement strand since the state indicates that standard 10.M.4 would not be assessable in 2001. There are no items mapped to this standard.
4) Range calculations exclude items scoring 0 for content centrality.

Level of Challenge and Balance (State Map)

Standard/ # of Items	Comments
Number Sense & Operations: 9 items 22.0%	All items are Level 1 (6 items) or Level 2 (1 item), with 2 items not scorable for level. Eight of the 9 items are multiple-choice, with the remaining item being open-response. One item assesses an eighth-grade standard. This item set, while at an appropriate level given the stakes of this test, tends to assess the less cognitively demanding aspects of these standards. Items emphasize numeric and symbolic manipulation at the expense of number sense and computational skills to solve problems, making this the least rigorous of the item sets. While the state map shows all 4 standards being assessed by at least one item, both items mapping to standard 10.N.4 were judged not to address the content of these standards.
Patterns, Relations, & Algebra: 10 items 24.4%	One item is Level 3, 4 items are Level 2, and 4 items are Level 1, with 1 item not scorable for level. Three of the 10 items are open-response and 2 of the items are short-answer, with the remaining 5 items being multiple choice. This mix of item formats contributes to an enhanced level of challenge for this item set. Two items assess eighth-grade standards. Five of the 8 standards are assessed by at least one item, but those standards not assessed (10.P.2, 10.P.3, and 10.P.6) contain important content.
Geometry: 10 items 24.4%	All items are Level 1 (2 items) or Level 2 (7 items), with 1 item not scorable for level. Two of the 10 items are short-answer, with the remaining items being multiple-choice. This item set contains no open-response items. While the level of challenge is less demanding than for Patterns, Relations, and Algebra, it still tends to be appropriate. Three items assess eighth-grade standards. With 10 of the 11 standards being assessable in 2001, the state map shows 6 of these standards being assessed by at least one item. However, the item mapping to 10.G.5 was judged not to address the content of these standards. Both level of challenge and balance are impacted by these gaps.

Standard/ # of Items	Comments
Measurement: 4 items 9.7%	All items are Level 1 (1 item) or Level 2 (3 items). Three of the items are multiple-choice, and 1 item is open-response. While the level of challenge is less demanding than for Patterns, Relations, and Algebra, it still tends to be appropriate. Three of the problems are presented in the context of a real-world problem. The level of challenge of this item set is tempered somewhat by the fact that students are provided with a Mathematics Reference Sheet that includes formulas. Each of the items maps to a grade 10 standard. Balance is an issue since the state map shows all 4 items mapping to the same standard (10.M.1). Three of the 4 items require area calculations and 2 of the 4 items require circumference calculations. Students are not required, for example, to find surface area or volume. This item set does not represent the balance of knowledge and skills represented in the standards.
Data Analysis, Statistics, & Probability: 8 items 19.5%	One item is Level 3, 5 items are Level 2, and 1 item is Level 1, with 1 item being not scorable for level. Seven of the 8 items are multiple-choice, with the remaining item being open-response. Seven of the 8 items assess eighth-grade standards, leaving 2 of the 3 tenth-grade standards not assessed. This raises concerns with respect to both level of challenge and balance. This item set does, however, contain a good mix of items requiring students to address the 3 areas of data analysis, statistics, and probability. In addition, the items include a variety of types of graphical and tabular data displays.
TEST AS A WHOLE: 41 items 100%	Test items are predominantly Level 1 (14 items) and Level 2 (21 items). Two items are rated as Level 3, and 5 items were not scorable. The level of challenge is generally appropriate. This test is more challenging than a number of other grade 10 tests but is still fairly and appropriately challenging. While it is the intent of this test to assess standards from earlier grade levels, the number of items mapping to grade 8 standards is substantial—13 items, including 1 open- response item and 1 short-answer item. This accounts for over 30% of the test items. The distribution of items across the various strands is as follows: Number Sense and Operations (22%), Patterns, Relations, and Algebra (24%), Geometry (24%), Measurement (10%), and Data Analysis, Statistics, and Probability (20%).

Level of Challenge and Balance (Achieve Map)

Standard/ # of Items	Comments
Number Sense & Operations: 6 items 14.6%	Five items are Level 1, with 1 item not scorable for level. All 6 items are multiple-choice. One item assesses an eighth-grade standard. This item set, while at an appropriate level given the stakes of this test, tends to assess the less cognitively demanding aspects of these standards. Items emphasize numeric and symbolic manipulation at the expense of number sense and computational skills to solve problems, making this the least rigorous of the item sets. A lack of short-answer or open-response items also diminishes the level of challenge of this item set. One standard (10.N.4) is not assessed since one of the items originally mapped to it was re-mapped and the other item was judged not to address the content of these standards.

Standard/ # of Items	Comments
Patterns, Relations, & Algebra: 14 items 34.1%	One item is Level 3, 6 items are Level 2, and 6 items are Level 1, with 1 item not scorable for level. Four of the 14 items are open-response and 2 of the items are short-answer, with the remaining 8 items being multiple choice. This mix of item formats contributes to an enhanced level of challenge for this item set. Four items assess eighth-grade standards. Six of the 8 standards are assessed by at least one item, but those standards not assessed (10.P.2 and 10.P.3) contain important content.
Geometry: 9 items 22.0%	All items are Level 1 (2 items) or Level 2 (6 items), with 1 item not scorable for level. Two of the 10 items are short-answer, with the remaining items being multiple-choice. This item set contains no open-response items. While the level of challenge is less demanding than for Patterns, Relations, and Algebra, it still tends to be appropriate. Three items assess eighth-grade standards. Both the level of challenge and balance of this item set are impacted by the fact that only 4 of the 10 standards identified by the state as assessable in 2001 are indeed assessed by this set of test items.
Measure-ment: 4 items 9.7%	All items are Level 1 (1 item) or Level 2 (3 items). Three of the items are multiple-choice, and 1 item is open-response. While the level of challenge is less demanding than for Patterns, Relations, and Algebra, it still tends to be appropriate. Three of the problems are presented in the context of a real-world problem. The level of challenge of this item set is tempered somewhat by the fact that students are provided with a Mathematics Reference Sheet that includes formulas. Three of the 4 items map to grade 10 standards, while the remaining item maps to an eighth-grade standard. Two of the 3 standards identified by the state as assessable in 2001 are indeed assessed, with 10.M.3 being the only eligible standard not assessed. Three of the 4 items require area calculations and 2 of the 4 items require circumference calculations. Students are not required, for example, to find surface area or volume. This item set does not represent the balance of knowledge and skills represented in the standards.
Data Analysis, Statistics, & Probability: 8 items 19.5%	One item is Level 3, 5 items are Level 2, and 1 item is Level 1, with 1 item being not scorable for level. Seven of the 8 items are multiple-choice, with the remaining item being open-response. Seven of the 8 items assess eighth-grade standards, leaving 2 of the 3 tenth-grade standards not assessed. This raises concerns with respect to both level of challenge and balance. This item set does, however, contain a good mix of items requiring students to address the 3 areas of data analysis, statistics, and probability. In addition, the items include a variety of types of graphical and tabular data displays.
TEST AS A WHOLE: 41 items 100%	Test items are predominantly Level 1 (15 items) and Level 2 (20 items). Two items are rated as Level 3, and 4 items were not scorable. The level of challenge is generally appropriate. This test is more challenging than a number of other grade 10 tests but is still fairly and appropriately challenging. While it is the intent of this test to assess standards from earlier grade levels, the number of items mapping to grade 8 standards is substantial—16 items, including 1 open- response item and 1 short-answer item. This accounts for almost 40% of the test items. The distribution of items across the various strands is as follows: Number Sense and Operations (15%), Patterns, Relations, and Algebra (34%), Geometry (22%), Measurement (10%), and Data Analysis, Statistics, and Probability (19%).