| **Background Information** |
| --- |
| Content/Course | Algebra I |
| Unit | Unit 1: Relationships Between Quantities and Reasoning with Equations |
| Essential Questions/Enduring Understandings Addressed in the Lesson | Essential Questions* **What characteristics of problems would determine how to model the situation and develop a problem solving strategy?**
* *What characteristics of problems would help to distinguish whether the situation could be modeled by a linear or an exponential model?*
* *When is it advantageous to represent relationships between quantities symbolically? numerically? graphically?*

Enduring Understandings* **Mathematics can be used to find solutions to real world problems and can be used to communicate solutions to stakeholders.**
* **Situations that have numbers or objects that repeat in predictable ways can be described using generalizations based on mathematics.**
* **Relationships between quantities can be represented symbolically, numerically, graphically and verbally in the exploration of real world situations.**
* **Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.**
	+ *When analyzing real problems that are math related, it is useful to look for patterns that would indicate that a linear or exponential model might be used to represent the situation.*
* **There are multiple models to represent a given real world relationship.**
 |
| Standards Addressed in This Lesson | **A.CED.2**Create linear and exponential equations in TWO variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.**N.Q.3**  Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| Lesson Title  | Exponential Relationships |
| Relevance/Connections | This lesson connects to prior learning of linear relationships and to future learning of graphing exponential relationships/functions as well as to geometric and arithmetic sequences.Exponential relationships also relate to many science topics. Specific options to include those science connections in this lesson will depend on the science course(s) in which the students in the class are currently enrolled. Consider working with science teachers in your school to determine if there is a natural connection to a science topic relevant to your students.  |
| Student Outcomes  | The student will:* understand that a quantity that grows or decays by a fixed ratio at regular intervals can be represented by an exponential relationship.
* write exponential relationships from patterns given numeric or verbal representations.
 |
| Summative Assessment(*Assessment of Learning*)  | *What evidence of student learning would a student be expected to produce to demonstrate attainment of this outcome?** Given a pattern in numeric and/or verbal form, students will identify the pattern as exponential or non-exponential, and justify their choice using properties of exponential relationships.
* Given a pattern in numeric and/or verbal form, students will be able to write an exponential expression or equation to model the given pattern.
 |
| Prior Knowledge Needed to Support This Learning(*Vertical Alignment*) | Grade 8 contains extensive work and focus on students’ understanding and ability to represent linear relationships in symbolic, numeric, and verbal representations. Students in 8th grade also have experience with functions, but might need review of function notation for Part III of “Saving Pennies” activity.  |
| Method for determining student readiness for the lesson | *How will evidence of student prior knowledge be determined?*From prior lessons in this unit on linear patterns, teachers should have data regarding student understanding of linear and non-linear relationships. This data can be combined with the pre-assessment activity, to determine student readiness and needs for remediation or enrichment.*What will be done for students who are not ready for the lesson?*Teachers can consider use of MSDE online Algebra/Data Analysis course Unit 4 Lesson 6 for students still struggling with linear vs non-linear relationships.<http://msde.mdk12online.org> Username: AlgebraPassword: student* Teachers may consider differentiating the groups by ability when using Activity 2 (see page 9), providing more scaffolding on worksheet and/or challenging advanced learners to determine a symbolic form for the model.
* Teachers may also consider grouping based on student learning preferences, or problem solving approaches based on this or other pre-assessment data.
 |
| Common Misconceptions**(See Algebra I Unit 1, unit plan, for strategies for dealing with the misconception.)** | Incorrect use of the mathematical concept. *Student may believe that anything that grows rapidly is exponential growth. Example, increases in enrollment growing rapidly over a series of years might be referred to as “growing exponentially” but is really just following a steep linear curve.* |

| **Learning Experience** |
| --- |
| ***Standards for******Mathematical Practice******(SMP)*** | ***Component*** | ***Details*** |
| **SMP#3 Construct viable arguments and critique the reasoning of others.** Students will be critiquing the reasoning of their partner. Take time to provide guidance in this process, set expectations for what you will be listening for in their conversations, and/or model the process to ensure that students are maximizing this opportunity. Consider providing the description of this practice to the students. | ***Hook/ Pre-Assessment*** | **Materials needed** * “Follow the Pattern” handout (page 14)
* “Relationships” handout (page 15)
* “Linear vs. Non-linear Relationships Pre-Assessment Record” recording sheet (page 16)

 **Preparation***Directions and items to complete this task will need to be printed, duplicated, cut, and placed into envelopes in preparation for this activity. Consider copying items on one color and their matches on another color. An alternative could be to use clear sleeve covers with dry erase markers for students to write matches, rather than cutting and sorting***Implementation**:*The pre-assessment activity could be completed at the beginning of the lesson or the day before. The purpose of the pre-assessment is to determine if students have retained the necessary prior knowledge needed to complete this lesson. Specifically students need to extend a pattern, distinguish between linear and non-linear relationships, and write a linear relationship in symbolic form.* *Carefully planned observational assessment is needed to identify any students who are not yet ready for the lesson. On page 16 you will find a tool that you may elect to use for making note of your observations.* **Facilitating the activity** * Group students into pairs (for the purpose of this lesson description, we will call one student “Student A” and the other “Student B”).
* Distribute the “Relationships” envelope to Student A in each pair.
* Distribute the “Follow the Pattern” envelope to Student B in each pair.

*Note: The “Follow the Pattern” exercise gets a bit more difficult as the chain progresses. This may allow for teachers to determine students’ level of understanding of how to complete a pattern based on where students have difficulty. Also note that the* 1, 1, 2, 3, 5, 8, 13, …, and the 1, 1.618, 2.618, 4.236, 6.854, 11.09, … *patterns are special patterns that could lend themselves to further exploration.** Instruct all students to work independently to complete the activity in their envelope.
* When both students in a pair complete their respective activities, instruct them to check the work of their partner.
* Student pairs should provide their partner with viable arguments as to why they completed the activity the way they did. Arguments should be justified through the use of mathematical principles.
* As a whole class debrief, ask various pairs to share their arguments with the class for a few selected problems.

*Possible questions to ask* * “Why does this relation belong in the linear/non-linear group?”
* “How did you decide that this relation is linear/non-linear?”
* “How did you determine the next terms in this pattern?”
* Each pair should then choose one of the linear relationships from student A’s set and represent the given relationship symbolically, numerically, verbally, and graphically.
* Ask pairs to share their work.

**Differentiation** *Include the possible questions with the sort activities to prompt student thinking. This is especially useful if students are not yet familiar with classroom discourse routines.***UDL Connections**The grouping used for this activity provides a connection to UDL Principle III Provide Multiple Means of Engagement. Checkpoint 8.3 Foster Collaboration and Communication<http://www.udlcenter.org/aboutudl/udlguidelines/principle3#principle3_g8> |
| **SMP#1 Make Sense of Problems and Persevere in Solving Them** The Motivation activity provides an opportunity to concentrate on SMP #1, by allowing students to determine their own path in answering the motivating question.**SMP #4 Model with Mathematics** This piece of the lesson could be a student-centered investigation; it might make more sense for this to be a whole class discussion facilitated by the teacher. Even if this is teacher-facilitated, the opportunity to build students’ ability to “**Model with Mathematics**” (MP 4) should not be dismissed. The teacher may ask the guiding questions, but should allow the students to take charge of how they want to model the problem. What geometry might they be able to use? Can they set up an equation?  | ***Motivation*** | **Materials Needed*** box
* enough pennies to fill the box

**Preparation** *In preparation for this lesson, obtain a box and collect, or ask students to collect, pennies to fill the box. Take the time to actually fill the box to allow student a visual of the determined number of pennies.***Implementation*** Ask the class, “How many pennies will it take to fill this room?”
* Allow students to discuss and record guesses on the board.
* Include discussions about level of accuracy needed to discuss the result.
	+ How many pennies will it take to fill a cube with volume of 1 cubic foot? (note: other objects can be substituted such as a tissue box or gallon milk jug.)
	+ How many boxes of pennies to fill room?
	+ Do we need to provide an answer to the nearest penny to answer this question?

**Extension**Ask students to determine how much money is in the box, room, and possibly a larger area. Again, take time to discuss the level of accuracy needed to reflect the solution(s). (Do we need to report to a fraction of a penny, nearest penny, dollar, hundred dollars, etc.?)**UDL Connections**This activity provides an example of how UDL principle III: Provide Multiple Means of Engagement can be represented in a lesson. More specifically this activity adheres to Guideline 7: Provide options for recruiting interest ,Checkpoint 7.2: Optimize relevance, value and authenticity.<http://www.udlcenter.org/aboutudl/udlguidelines/principle3#principle3_g8> |
| **SMP#1 Make sense of problems and persevere in solving them.** While students can connect to a variety of Standards for Mathematical Practice in this activity, try to focus on SMP#1. It is natural to want to point students in the correct direction, but the class has just considered a similar situation and students should be given a chance to choose a similar method or devise their own plan. Encourage students to demonstrate the following characteristics of mathematically proficient students. Providing a rubric for this might be a way to informally capture student progress towards proficiency.* plan a solution pathway rather than simply jumping into a solution attempt
* consider analogous problems (such as the motivational activity)
* monitor and evaluate their progress and change course if necessary
* explain correspondences between verbal descriptions and numeric representations
* search for regularity or trends
* For those students who are struggling, you might provide concrete objects or pictures to help conceptualize and solve a problem
* Continually ask, “Does this make sense?”
 | ***Activity 1***  | **Materials** * Part I of the “Saving Pennies” handout (pages 17)

**Implementation*** Place students in groups based on pre-assessment data. These groups could be based on ability, observed methods of solving problems, student learning preferences, or other grouping techniques.
* Groups should complete Part I of the “Saving Pennies” handout. Note that no scaffolding is provided for the students. Challenging students to complete unscaffolded problems provides opportunities to build their proficiency with the Standards for Mathematical Practice. At this point, you are not looking for correct answers, but rather the realization that doubling (tripling, etc) grows much faster than adding a specific amount each week.

Possible teacher questions to facilitate (only use these questions if needed):* For question #2: What if tomorrow I double the number of pennies you already have in your box?
* Do you want me to do this again the next day? Why? What would happen?

Possible extension question for students who need enrichment and/or are moving faster:* Do you think it is reasonable to ask students to carry around a box of one cubic foot? *Hopefully students will consider the weight of the box in their response.*

**Summary** After groups have completed problems 1-3 in Part I, the following questions will prompt classroom discussion to ensure students grasp the concept that exponential relationships grow or decay by a fixed ratio at regular intervals.* Have groups share the strategies they chose to fill their box more rapidly. *Expect that groups will have doubled each day, tripled each day, or something of the like. This might be an opportunity to critique the reasoning of others (SMP#3). If a group elects to quadruple their pennies each day, would this be considered an unrealistic task?*
* What are the differences between the given situation and your new situation? *Answers will vary depending on the response to question 2 The hope is that students will create a pattern that leads to an exponential model.*
* What kind of function models the situation described at the top of Part I of the “Saving Pennies” handout? Why? *Linear, constant rate of change.*
* What if we added 100 pennies the first and second days, but then added 500 pennies the third day? Is this still the same function? *No, this is not a constant rate of change. The visual display below may be helpful for some students.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Day*** | *1* | *2* | *3* |  |
| ***Amount*** | *100* | *200* | *700* |  |

* What if our pennies were collected in this way?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Day** | 1 | 2 | 5 | 6 |
| **Amount** | 100 | 200 | 300 | 400 |

* *This is increasing at a constant interval (each day), but not by a constant amount, so this is not a constant rate of change, and therefore not linear.*
* What is different about your new situation? *Multiplied by a constant amount each day. In other words, it increases at a constant ratio at a constant interval.*
* The function that models this behavior is an exponential function. An exponential function grows or decays at a constant ratio over a constant interval. Show the example below (which hopefully was produced by one of the groups).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Day** | 1 | 2 | 3 | 4 | 5 |
| **Amount** | 100 | 200 | 400 | 800 | 1600 |

* What if we doubled the number of pennies the first and second days, but then added 500 pennies the third day? Or quadrupled the number of pennies the third day? Is this still the same function? *No, this is not a change of a constant ratio over a constant interval. The visual display below may be helpful for some students.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Day*** | *1* | *2* | *3* |  |
| ***Amount*** | *100* | *200* | *700* |  |

* What if our pennies were collected in this way?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Day** | 1 | 2 | 5 | 6 |
| **Amount** | 100 | 200 | 400 | 800 |

*This is increasing at a constant ratio (doubling, ratio of 2), but not at a constant interval (each day).***Formative Assessment/***Evidence of Student Learning* * Instruct students to complete a Frayer model for “linear relationship.” (see attached)
* Instruct students to complete a Frayer model for “exponential relationship.”(see attached)
* Use student responses to collect evidence of student understanding of the concepts of linear and exponential relationships.
 |
| **SMP#3 Construct viable arguments and critique the reasoning of others.** While students can connect to a variety of Standards for Mathematical Practice in this activity, try to focus on SMP#3. Encourage students to demonstrate the following characteristics of mathematically proficient students while forming their decisions.*Characteristics** understand and use stated assumptions, definitions, and previously established results in constructing arguments
* justify their conclusions, communicate them to others, and respond to the arguments of others
* read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments

Providing a rubric for this might be a way to informally capture student progress towards proficiency. | ***Activity 2*** | **Materials*** Part II of “Saving Pennies” handout

**Implementation*** Groups should complete Part II of the “Saving Pennies” handout. Note that no scaffolding is provided for the students. Challenging students to complete unscaffolded problems provides opportunities to build their proficiency with the Standards for Mathematical Practice. Though, based on student needs in the class, providing a structure might be a good way to differentiate.

Opportunity to differentiate* Those struggling with this concept might need a table to scaffold
* Students who already demonstrated understanding of this concept might be ready for a challenge to write an expression/equation for the situation.
* Teachers should consider grouping students by learning preference or usual method of problem solving (do you have several students who like to look at problems graphically, or others who often use tables, etc.)

**Summary*** Display a visual comparison of the two relationships described on Part II of the “Savings Pennies” handout. (i.e. tables or graphs). If student groups created such displays, ask various groups to share.
* Discuss in generalities the shape of the exponential curve. (we are not having students graph an exponential function at this point)

**Formative Assessment**Chose one of the two contexts below, or create a context that is more relevant to your students. Ask students to decide between the two scenarios in the context and justify their choice.Context AYou didn’t realize that your parents do not have an unlimited texting plan on the family cell phones and you’ve racked up $340 of texting charges on your parents’ bill. Would you rather pay them back $8 each week, or at the rate of 1/8 of the remaining balance each week? Justify your choiceContext BAn exterminating company has treated your house for the infestation of stink bugs in your house. Would you choose the treatment that promotes that the stink bugs will decrease by 10 bugs each hour or by 1/10 of the remaining bugs each hour? Justify your choice. |
| **SMP#8 Look for and express regularity in repeated reasoning.**Students can connect to a variety of Standards for Mathematical Practice in this activity, try to focus on SMP#8. The accompanying worksheet provides some structure to the process of relating the recursive representation of the pattern to the explicit representation of the pattern. Though it is difficult to see students struggle, take the time to allow them to process what they are doing and ask questions of themselves and each other to discover the pattern and make generalizations. Encourage students to demonstrate the following characteristics of mathematically proficient students. *Characteristics* * Notice if and when calculations are repeated.
* Look for general methods and shortcuts.
* Notice regularity in the way values are rearranged or manipulated to simplify and provide structure.
* Maintain oversight of the process while attending to the details.
* Continually evaluate the reasonableness of their intermediate results.
 | ***Activity 3***  | **Materials** * Part III of the “Saving Pennies” handout (pages 17)

**Implementation*** Groups should complete Part III of the “Saving Pennies” handout. Part III requires students to build the symbolic form of the relationships they’ve examined in Parts I and II. Students will first look at the recursive relationship relating the current term to the previous term. Students will then build an explicit relationship, relating the independent variable (in this case the number of days) to the dependent variable (in this case, total number of pennies).
* Using a Jigsaw approach, form new groups and have students compare answers. Complete a whole class final debrief of problems 1-10 from Part III of the “Savings Pennies” handout.

**Extension** * Assign either problem 4, 5 or 6 to the student groups.
* The group members should create a scatter plot of their assigned problem on the attached grid. ( page 24) After completing the scatter plot, instruct students to sketch a curve which contains the points on the scatter plot.
* Groups should display their scatter plots.
* Ask students to share their observations about the behavior of the displayed curves. Note: This is simply to make connections; students are not expected to be able to graph an exponential function at this point.

**Formative Assessment**Provide students with several additional situations for which they will write an exponential expression. This could be done on individual white boards, with students at boards around the room, or simply on their paper for teacher observation. |
|  | ***Closure*** | **Materials Needed*** 4 Corners Signs (attached)
* Index cards displaying verbal, numeric, visual or symbolic representations of relationships ( enough for each student in your largest class)

**Implementation**This activity is a 4 Corners activity. * Post 4 signs around the room. (signs are attached)
	+ Exponential
	+ Linear
	+ Other
	+ Not sure
* Give each student a card which displays a verbal, numeric, visual, or symbolic representation of a relationship.
* Instruct students to stand next to the sign which displays the label that describes the relationship on their card.
* Students standing together should compare the relationships displayed on their cards and discuss what was it about the relationship that made them believe that they moved to the correct location.
* Provide assistance to students who move to the “Not Sure” corner.
* Ask one student from each corner to share the highlights of their corners’ discussion.

**UDL Connection**This activity adheres to UDL Principle II: Provide Multiple Means for Action and Expression, Principle 4: Provide Options for Physical Action in that it gives students an option of showing what they know be using physical movement.  |

| **Supporting Information** |
| --- |
|  | ***Details*** |
| Interventions/Enrichments* Special Education/Struggling Learners
* ELL
* Gifted and Talented
 | **Special Education/Struggling Learners** |
| **ELL**Provide explicit vocabulary instruction on…* Constant, Rate, Interval, decay,
 |
| **Gifted and Talented**Students could examine more deeply the relationships and patterns within the Fibonacci Sequence, Fibonacci Cascade, and the Golden Ratio. A particularly interesting discovery is that the Golden Ratio can be used in a variety of ways to create the Fibonacci sequences. |
| Materials | * Relationships– *will need to be cut and put in envelope with directions.*
* Follow the Pattern *– will need to be cut and put in envelope with directions.*
* Linear vs. Non-Linear Pre-assessment Record
* “Saving Pennies” handout
* 1 square foot box, tissue box, gallon milk jug, or similar container
* pennies
 |
| Technology | * Optional

<http://msde.mdk12online.org> Username: AlgebraPassword: student* Optional graphing technology
 |
| Resources(must be available to all stakeholders) | The two sites below are good places to get more information about the Fibonacci Sequence and the Fibonacci Cascade as they relate to the Golden Ratio. In the second link, be sure to scroll down a bit to the part about the Golden Cascade to relate to one of the patterns in the “Relationships” pre-assessment task.<http://www.goldennumber.net/><http://www.goldenmeangauge.co.uk/fibonacciguage.htm> |

**Follow the Pattern**

**Directions:** Examine the cards in your envelope. Each card has two lines on it. The **second** line is a pattern for which you need to determine the next two terms. Remember to consider adding, subtracting, multiplying, or dividing when comparing one term to the next in order to determine your pattern. Once you’ve determined the next two terms in the given pattern, locate the card that has these two terms as the **first** line, and then match them up. Continue by completing the pattern on the second line of this matching card. When you are finished, you will have a chain of patterns.

|  |  |  |  |
| --- | --- | --- | --- |
| 25, 37, 49, 61, … | 12, 48, 12, 3, , … | 8575, -60025, , 7, , … | -17, -38, 2, ,  |
| 73, 8581, 27, 9, 3, … | 27, 40, 53, 66, … | , , -10, -15, , … | -3, , , ,  |
| 1, 10, , 11, , … | 79, 92-25, 175, -1225, … | , 88, 67, 46, 25, 4, … | 2, 1, , , , … |

**Relationships**

**Directions:** Your envelope contains two cards larger in size: linear and not linear. These will be the categories in which you will group the remaining cards. Examine the rest of the cards in your envelope. Each has a pattern that you must place in the linear or not linear category.

|  |  |
| --- | --- |
| **Linear** | **Not Linear** |
| 25, 37, 49, 61, … | 10, , 11, , … | 81, 27, 9, 3, … | -25, 175, -1225, … |
| , , 7, , … | 88, 67, 46, 25, 4, … | , -10, -15, , … | 1, 4, 9, 25, 36, … |
| , , ,  | , 2, ,  | 26, 28, 32, 38, 46, … | 1, 2, 6, 24, 120, … |
|  |  | 1, 1, 2, 3, 5, 8, 13, … | 1, 1.618, 2.618, 4.236, 6.854, 11.09, … |

**Linear vs. Non-linear Relationships**

**Pre-Assessment Record**

S = Needs Support P= Proficient A = Advanced

|  |  |  |  |
| --- | --- | --- | --- |
| **Student Name** | **Identify Next Terms** | **Identify Linear** | **Identify Non-Linear** |
| **S** | **P** | **A** | **S** | **P** | **A** | **S** | **P** | **A** |
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**Saving Pennies**

**Part I:**

To raise money for the end-of-year dance, your class officers have asked you to participate in a fundraiser where you fill a one cubic foot box with pennies that are donated by friends and family. You’ve set a goal to collect 100 pennies every day.



1. How long will it take you to fill the box with pennies?
2. Describe a faster way to fill the cubic foot box.
3. Using your new strategy, how long will it take to fill the box? How long will it take to fill the room?

**Saving Pennies**

**Part II:**

Your parents would like to save money for you as a graduation gift. They have given you the two options below. Which option would you choose? Justify your choice.

* Option 1: They would save 1 penny the first week. For each of the following weeks they would save double the number of pennies as the week before.
* Option 2: They would save $100 each week.

**Saving Pennies**

**Part III:**

As your fund-raising plan, you decide to put 5 pennies in your box to encourage friends and family to contribute and set a goal to double your total number of pennies each day.

1. Why would you choose to double your pennies each day, rather than add 100 pennies each day?
2. What is the constant ratio that will define your exponential model?
3. Write a rule for this sequence to determine the total number of pennies in the next day.
4. Complete the table below so that you can model your plan for the class officers.

|  |  |  |
| --- | --- | --- |
| **Number of days (d)** | **Total number of pennies (in expanded form) - T(d)** | **T(d)** |
| 0 | T(0)=5 |  $T(0)=(5)∙2$ |
| 1 | $$T\left(1\right)=T\left(0\right)∙2$$ $=(5)∙2$ |  $T(1)=(5)∙2$ |
| 2 | $$T\left(2\right)=T\left(1\right)∙2$$ $=\left(5∙2\right)∙2$ $=(5)∙2^{2}$ |  $T(2)=(5)∙2$ |
| 3 | $$T\left(3\right)=T\left(2\right)∙2$$ $=\left((5)∙2^{2}\right)∙2$ $=(5)∙2^{3}$ |  $T(3)=(5)∙\\_\\_\\_\\_$ |
| 4 | $$T\left(4\right)=T\left(3\right)∙2$$ $=\left((5)∙2^{3}\right)∙2$ $=$ |  $T(4)=(5)∙\\_\\_\\_\\_\\_\\_$ |
| 5 |  | $T(5)=$  |
| d |  |  |

1. What if you start with 8 pennies and decide to try to triple your number of pennies each day? Write a model to represent this situation.

|  |  |  |
| --- | --- | --- |
| **Number of days (d)** | **Total number of pennies (in expanded form) - T(d)** | **T(d)** |
| 0 | T(0)=8 |  $T(0)=(8)∙3$ |
| 1 | $$T\left(1\right)=T\left(0\right)∙3$$ $=(8)∙3$ |  $T(1)=(8)∙3$ |
| 2 | $$T\left(2\right)=T\left(1\right)∙3$$ $=\left(8∙3\right)∙3$ $=(8)∙3^{2}$ |  $T(2)=(8)∙3$ |
| 3 | $$T\left(3\right)=T\left(2\right)∙3$$   |  $T(3)=$ |
| d |  |  |

1. Is the above situation realistic? What if you try a ratio of 1.5? What would that mean in terms of daily increase of pennies? Write a model for this situation choosing your own initial number of pennies.
2. What is the fastest way to fill your box while still being realistic? Try different combinations of initial number of pennies and ratios to arrive at your own plan.
3. You didn’t realize that your parents do not have an unlimited texting plan on the family cell phones and you’ve racked up $340 of texting charges on your parents’ bill. Write a model for paying your parents back at the rate of 1/8 of the remaining balance each week?
4. An exterminating company has treated your house for the infestation of stink bugs in your house. Write a model that represents the stink bugs decreasing by 1/10 of the remaining bugs each hour. Assume you have a thousand stink bugs in your home before the exterminator treats your house.
5. Assuming an initial value of “a” and common ratio of “b,” write an expression that will represent any exponential model.



**Linear**

**Relationship**



**Exponential Relationship**

