

This task was developed by high school and postsecondary mathematics and design/pre-construction educators, and validated by content experts in the Common Core State Standards in mathematics and the National Career Clusters Knowledge & Skills Statements. It was developed with the purpose of demonstrating how the Common Core and CTE Knowledge & Skills Statements can be integrated into classroom learning – and to provide classroom teachers with a truly authentic task for either mathematics or CTE courses.

TASK: STAIRWAY

TARGET COMMON CORE STATE STANDARD(S) IN MATHEMATICS:

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

TARGET STANDARDS FOR MATHEMATICAL PRACTICES

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

TARGET COMMON CORE STATE STANDARD(S) IN ELA/LITERACY:

RST.9-10.1 Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.

RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

RST.11-12.7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

RST.11-12.9 Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.

WHST.11-12.7 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

TARGET CAREER AND TECHNICAL EDUCATION (CTE) KNOWLEDGE & SKILLS STATEMENTS:

ACC01.01.05 Use appropriate formulas to determine measurements of dimensions, spaces and structures.

ACC01.01.06 Conceptualize a 3-dimensional form from a 2-dimensional drawing to visualize proposed work.

ACC02.01.02 Utilize vocabulary and visual cues in context of design and construction situations.
ACC05.01.01 Identify governmental regulations and national, state, and/or local building codes that apply to a given workplace/jobsite.
ACC05.01.02 Evaluate workplace/jobsite activities for compliance with governmental and other applicable safety regulations such as EPA and OSHA.
ACC10.02.01 Select tools, machinery, equipment, and resources that match requirements of the job.
ACPA06.01.01 Identify client requirements.
ACPA06.01.04 Learn to read and produce technical drawings, understanding the significance of each line in a drawing.
RECOMMENDED COURSE:
Algebra 1, Geometry; Integrated Math I, Integrated Math II; Applications in Design & Pre-Construction
ADDITIONAL INSTRUCTIONS:
This task should be completed over an extended period of time, including some time for measurements research of local building codes, and building a physical model.

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

About the Common Core State Standards in Mathematics

The Common Core State Standards (CCSS) for Mathematics are organized by grade level in grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. www.corestandards.org

About the Common Core State Standards in English Language Arts/Literacy

The Common Core State Standards (CCSS) for ELA/Literacy are organized by grade level in grades K–8. At the high school level, the standards are organized by 9-10 and 11-12 grade bands. Across K-12 there are four major strands: Reading, Writing, Speaking and Listening, and Language. The CCSS also include Standards for Literacy in History/Social Studies, Science, and Technical Subjects, with content-specific (Reading and Writing) literacy standards provided for grades 6-8, 9-10, and 11-12, to demonstrate that literacy needs to be taught and nurtured across all subjects. www.corestandards.org

About the Career Cluster Knowledge and Skill Statements

As an organizing tool for curriculum design and instruction, Career Clusters™ provide the essential knowledge and skills for the 16 Career Clusters™ and their Career Pathways. It also functions as a useful guide in developing programs of study bridging secondary and postsecondary curriculum and for creating individual student plans of study for a complete range of career options. As such, it helps students discover their interests and their passions, and empowers them to choose the educational pathway that can lead to success in high school, college and career. www.careertech.org/career-clusters/resources/clusters/architecture.html. Although not included in this template, all Clusters and Pathways have Foundational Academic Expectations and Essential Knowledge & Skills Statements, which, in some cases, overlap with the Common Core State Standards.

KEY TERMS

- Tread (depth), riser (height)
- Total rise, Total run
- Stringer, nosing

STAIRWAY – *The Task*

You have been commissioned to design a stairway for a custom home. You may choose to design either an outdoor entry or interior staircase. You will need to gather information regarding design, safety, and the utility of staircases.

1. Examine several stairways around your school building, your home, and/or your neighborhood. Record the measurements for a variety of stairs, including riser height, tread depth, total rise, and total run.
2. Describe the type of stairway you will design, the material(s) you will use, and provide all specifications and overall measurements of finished stairs and materials.
3. Research and list building codes specific to your community/county, which apply to your staircase design.
4. Do the stairs you measured in Question 1 meet the local code? If not, why is that? What happens when you build steps in an unacceptable manner (i.e. not to code)?
5. Use the relevant codes to write true mathematical equations/inequalities that reflect code requirements.
6. Graph the system of inequalities from question 4 and use the graph to show the acceptable range of stairway dimensions (tread depth and riser height). Identify all critical points and lines and give coordinates and equations on the graph, including intersection points of all boundary lines. Shade the feasible region and specify three reasonable ordered pairs in your graph that might be used for tread and riser measurements in your design.
7. Create (build, draw, render, etc.) a to-scale sketch and a physical model of your staircase.
8. Determine the overall slope of your model. Show the slope on a diagram of the side of a stringer. Explain what happens to the slope if riser height increases but tread depth stays the same and also what happens if tread depth increases and the riser height stays the same?

STAIRWAY – Possible Solution(s)

1. *Answers will vary based on measurements.* However for each stairway examined, measurements must include tread depth, riser height, total rise, and total run.
2. *Descriptions will vary based on design decisions.*
3. *Answers will vary based on local codes.* An example is provided here using Chicago codes: [City of Chicago Department of Buildings, Porch Design and Construction Guidelines – www.cityofchicago.org/content/dam/city/depts/bldgs/general/EZPERMIT/PorchGuidelinesMay2011.pdf](http://www.cityofchicago.org/content/dam/city/depts/bldgs/general/EZPERMIT/PorchGuidelinesMay2011.pdf)

These codes from the City of Chicago are relevant to the design:

- a. 10(13-160-300)(b) – *In occupancies other than institutional and assembly units the maximum height of a riser shall be eight inches, and the minimum width* of a tread, exclusive of nosing, shall be nine inches. The width* of a tread, including nosing, shall be not less than ten inches.*
- b. 10(13-160-300)(c) – *The height of two risers plus the width* of one tread shall equal not less than 24 inches or more than 27 inches.*
- c. 10(13-196-570)(f) – *The riser height and the tread width* of each flight of stairs shall be uniform.*
- d. 10(13-160-220)(b) – *Stairs and Corridors. All stairs and corridors required as exits shall be not less than 44 inches in width with the following exceptions.*

* In this Chicago code the word “width” is used to mean the depth of the tread rather than the width of the staircase.

4. *Answers will vary on the first two parts of this question.*

Sample response to the third part of this question: Stairs that may have originally been built to code may settle or been damaged in other ways, which might put them out of code compliance. If stairs are not built to code there may be an accident, making the owner/builder of the stairs liable for any injury or damages incurred.

5. Using the City of Chicago example:

Let t = tread depth in inches (horizontal measure),

r = riser height in inches (vertical measure)

$$0'' \leq r \leq 8''$$

$$t \geq 10'' \text{ (with nosing)}$$

$$t \geq 9'' \text{ (w/o nosing)}$$

$$24 \leq 2r + t \leq 27 \rightarrow 2r + t \geq 24$$

$$2r \geq 24 - t$$

$$r \geq 12 - \frac{1}{2}t$$

and

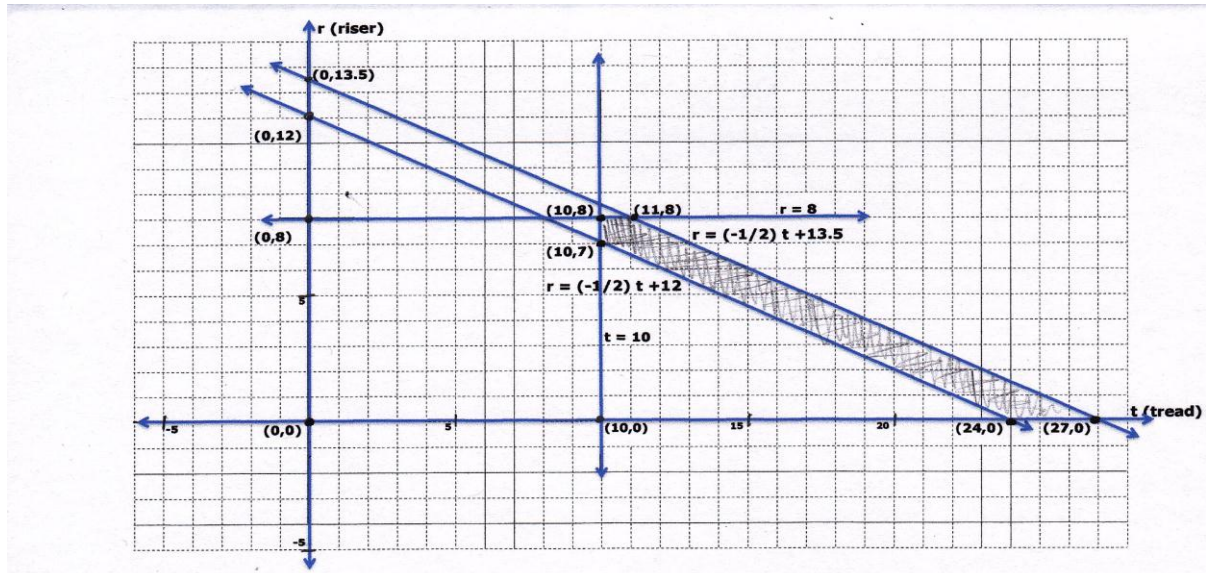
$$2r + t \leq 27$$

$$2r \leq 27 - t$$

$$r \leq 13.5 - \frac{1}{2}t$$

6. Graphs will vary depending on codes and inequalities found.

Using the Chicago example in the graph below: The points of intersection of the lines identified in question 5 are (10,7), (10, 8), (11,8), (27,0), and (24,0). The pentagonal region enclosed by lines contains all possible (tread, riser) ordered pairs. Fractional measures are common in stairs so ordered pairs, (tread, riser), may or may not be integral. The shaded region in the graph below includes all possible feasible ordered pairs for the stairs' tread and riser measurements.



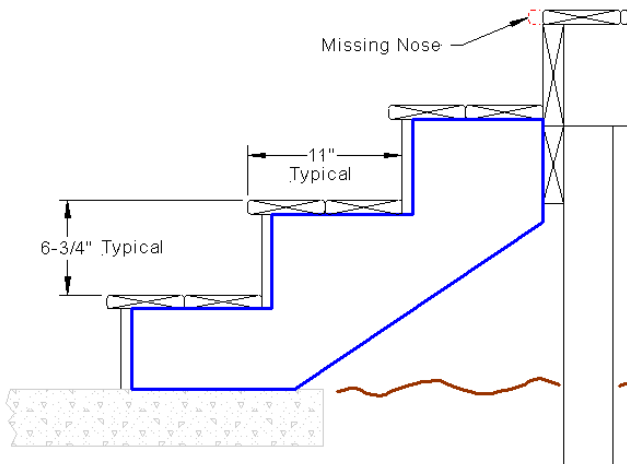
Examples of three reasonable points from this graph: (11, 7) (12,7) (13, 6)

7. Drawings and physical models will vary depending on the student's design decisions.

Possible resources:

- Additional City of Chicago code to use in design: 10(13-160-320)(a) - All stairways shall have walls, railings or guards on both sides and shall have handrails on both sides except as follows: (1) Stairs less than 44 inches wide may have a handrail on one side only. (2) Intermediate handrails, continuous between landings, shall be provided where required to provide a lateral distance between handrails not exceeding 88 inches.
- There are many how-to videos available on building decks and stairs. Here are just a few:
 - www.youtube.com/watch?v=PNeGtwOlfPw
 - www.askthebuilder.com/NH057 - Design the Perfect Set of Steps.shtml
 - www.youtube.com/watch?v=g7aa2gl0uvq
 - www.decks.com/Calculators/Stairs.aspx (Deck calculator)
 - www.carpentry-pro-framer.com/stairs.html (Stair-building instructions)
 - www.google.com/search?q=building+stairs&hl=en&biw=1126&bih=584&prmd=imvnsb&tbm=isch&tbo=u&source=univ&sa=X&ei=lhd1TPaVJ-zQiAK-t9CzAg&sqi=2&ved=0CG8QsAQ (Google images with examples of drawings)

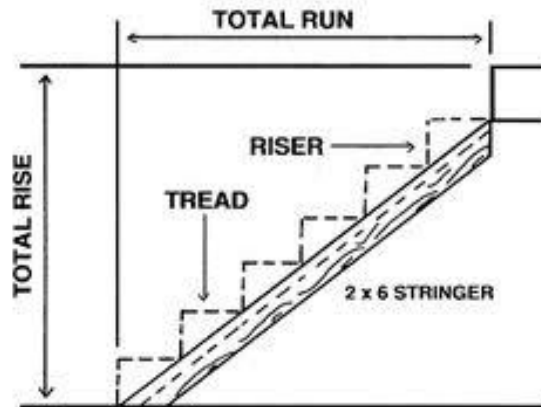
The drawing below is an example of those available on [Google images](#) with a search of “building stairs.”



Completed Stairs With Treads And Enclosed Risers

8. *Answers will vary based on individual design.* The overall slope of the model is Total Rise / Total Run (see diagram below). If the riser height increases but the tread depth stays the same, then the slope of the model will increase. If the tread depth increases and the riser height stays the same, the slope will decrease.

The following is an example taken from a [Google images](#) search of “building stairs.”



STAIRWAY – *Possible Extensions*

The extensions below represent potential ways in which mathematics and/or CTE teachers can build on the task above. All of the extensions are optional and can be used in the classroom, as homework assignments, and/or as long-term interdisciplinary projects.

1. Present a written proposal to your client. Include your scale drawing and an explanation of why you chose your design and use your graph and equations/inequalities to show how you know your staircase meets the code requirements.
2. Determine the cost associated with the project, using local pricing. Compare costs for various materials (e.g. treated lumber, specialty woods, composite decking, natural stone, concrete, steel, etc.)

STAIRWAY – Appendix: Alignment Ratings

The rating system used in the following charts is as follows:

3 EXCELLENT ALIGNMENT:

The content/performance of the task is clearly consistent with the content/performance of the Common Core State Standard.

2 GOOD ALIGNMENT:

The task is consistent with important elements of the content/performance of the CCSS statement, but part of the CCSS is not addressed.

1 WEAK ALIGNMENT:

There is a partial alignment between the task and the CCSS, however important elements of the CCSS are not addressed in the task.

N/A:

For Mathematical Practices a content rating does not apply.

In the charts **C = Content Rating** and **P = Performance Rating**

COLOR KEY

- **Black** = Part of CCSS/K&S Statement aligned to task
- **Gray** = Part of CCSS/K&S Statement *not* aligned to task

Task-to-Mathematical Practice Alignment Recording Sheet

Task Name	Aligned CCSS Mathematical Practice Standards	C	P	Alignment Comments	Task Comments
STAIRWAY	MP.1 Make sense of problems and persevere in solving them	N/A	3	For this task students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway. They must check the reasonableness of their solution, continually asking themselves, "Does this make sense?"	This is a multi-stage problem with real life applications and considerations. Students must identify measurements and lengths for the design project. They use both abstract reasoning and quantitative calculations.
	MP.2 Reason abstractly and quantitatively	N/A	3	This task involves quantitative relationships. It requires that students make sense of quantities and their relationships in the problem situation. They must attend to the meaning of the quantities and pay attention to units as they represent the quantities and measures in a table and then translate in a report to the client.	
	MP.3 Construct viable arguments and critique the reasoning of others	N/A	2	Students are required to justify their conclusions. There is no requirement to critique the reasoning of others	
	MP.4 Model with mathematics	N/A	3	This task is an application from everyday life requiring that the student create a geometric representation (model) that can replace the situation described in the prompt. Students must identify important quantities in the practical situation and use them to answer the questions posed. Students are required to explain their decisions and assumptions.	
	MP.5 Use appropriate tools strategically	N/A	3	Students must use the appropriate tools for measurement, design, and the creation of the physical model.	
	MP.6 Attend to precision	N/A	3	This task requires that students communicate precisely, organizing their information, as they show their mathematical thinking.	

Task-to-Common Core State Standards Alignment Recording Sheet

Task Name	Aligned CCSS Content Standards	C	P	Alignment Comments	Task Comments
STAIRWAY	A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	3	3	Students must write inequalities related to the building codes and consider the contextual limitations.	This task combines planning, reasoning, designing, calculating, and building a model
	A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	3	3	Question 6 of the task requires a graph of the system of inequalities found earlier in the task.	
	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	3	3	Since this standard asks for either linear OR exponential, this applies fully to the task, which only addresses linear graphs.	
	G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	3	3	This task has design as a key component.	
	6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	3	3	Students must understand and work with the riser to tread relationship as they determine the allowed lengths for each.	
	7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	2	3	Students are required to produce a scale drawing of their staircase design.	

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

Task-to-National Career Cluster Knowledge & Skills Statements Alignment Recording Sheet

Task Name	Aligned National Career Cluster Knowledge & Skills Statements	C	P	Alignment Comments
STAIRWAY	ACC01.01.05 Use appropriate formulas to determine measurements of dimensions, spaces and structures.	1	2	
	ACC01.01.06 Conceptualize a 3-dimensional form from a 2-dimensional drawing to visualize proposed work.	3	3	This only applies if students build a 3-D model.
	ACC02.01.02 Utilize vocabulary and visual cues in context of design and construction situations.	3	3	
	ACC05.01.01 Identify governmental regulations and national, state, and/or local building codes that apply to a given workplace/jobsite.	3	3	
	ACC05.01.02 Evaluate workplace/jobsite activities for compliance with governmental and other applicable safety regulations such as EPA and OSHA.	3	3	Identifying – and aligning the design to – local codes is a key element of this task.
	ACC10.02.01 Select tools, machinery, equipment, and resources that match requirements of the job.	3	3	This only applies if students build a 3-D model.
	ACPA06.01.01 Identify client requirements.	3	3	This task requires specific client specifications related to the design, safety and utility of stairs.
	ACPA06.01.04 Learn to read and produce technical drawings, understanding the significance of each line in a drawing.	3	3	This task requires a to-scale sketch and physical model of the project.