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Comparing the Common Core State Standards in Mathematics and the NAEP Framework

Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and the National Assessment of Educational Progress (NAEP) Framework.

Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering and Mathematics (STEM) coursework in college.

The National Assessment of Educational Progress (NAEP) Framework

NAEP is the only nationally representative and on-going assessment of what America's students know and can do in mathematics, reading, science, writing, the arts, civics, economics, geography, and U.S. history. NAEP assesses samples of students in grades 4, 8, and 12 in all 50 states and a number of U.S. territories using tests that are standardized to provide a common measure so that results can be compared over time and across states. NAEP results, in fact, were one reason for the development of CCSS; state-by-state comparisons on NAEP have revealed the disparity between states' assessments and NAEP and the wide variety of definitions of "proficient" used by states in their assessment systems. By providing a common metric of success, NAEP offers an honest benchmark for policymakers and educators to measure student success and make cross-state comparisons, much like the CCSS aim to do.

The NAEP assessments are based on frameworks that reflect current research on states' expectations in each subject area. These Frameworks are updated periodically, and any changes are documented carefully so the assessments can provide a clear picture of student achievement over time. The NAEP assessment Frameworks are designed to support test development, not to serve as a guide for curriculum or instruction. Thus, they focus on the content and skills that are deemed most important to assess at grades 4, 8 and 12.

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Based on the process used to develop the Frameworks, and the general high level of regard for NAEP by policymakers and educators alike, the NAEP Framework was an important resource for the developers of the CCSS in mathematics.

Achieve's Analysis

Achieve has analyzed the CCSS and the NAEP mathematics Framework to determine how they compare in terms of **rigor, coherence, and focus**. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; the standards should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the CCSS and the NAEP Framework, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

Major Findings

- ☑ Students who successfully master the CCSS will be well prepared for the NAEP exam in grades 4, 8 and 12. The CCSS and the NAEP Framework describe similar levels of rigor, with only minor differences in expectations described by the end of grade 8 and end of high school.
- ☑ The CCSS and NAEP Framework are similarly focused. The CCSS provide more coherence by providing more precise and clearer content expectations at each grade level and progressions of learning across the grades whereas the NAEP Framework focuses only on grades 4, 8 and 12.

Detailed Findings

Rigor

Overall, the two documents describe expectations of comparable rigor, despite a few differences. The NAEP mathematics Framework describes the key areas of mathematics that will be covered in the assessment (e.g. number properties and operations, measurement and geometry), and indicates the relative importance of each area for each grade level (4, 8 and 12) tested. It also indicates the level of mathematical complexity that will be assessed for each area in each grade.

- ☑ **Elementary grades:** The CCSS describe more rigorous expectations for students at the end of grade 4 than the NAEP Framework does. For example, both documents expect students in grade 4 to be fluent at adding, subtracting, and multiplying with whole numbers; to be able to apply place value; and to be able to classify simple two-dimensional geometric figures. The CCSS, however, also expect students to be able to multiply fractions by whole numbers. In addition, the CCSS expect students to understand multiplication algorithms through applications of place value and some properties of operations. As a result, students who have mastered the K-4 CCSS expectations should be well prepared for the 4th grade NAEP exam.

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Middle grades: Both the CCSS and the NAEP Framework have similar expectations for students by the end of 8th grade—both focus on knowledge of algebraic expressions, equations, and functions. Both expect students to be able to work effectively with the symbolic manipulations necessary to transform linear expressions and solve linear equations, which are hallmarks of beginning algebra. Differences are few. For example, the CCSS put more emphasis in the middle grades on geometric properties and using functions to model relationships between quantities than the NAEP Framework does. The NAEP Framework includes exponential growth and decay in grade 8. Overall, students who have mastered the K-8 CCSS expectations should be well prepared for the 8th grade NAEP exam.

High school: Expectations for high school students are quite similar, with only a few differences. When looking at the college- and career-ready expectations (those standards without a (+) in the CCSS and those NAEP objectives without a "*"[1]), the CCSS and NAEP Framework are well aligned with minor differences. While the NAEP assessment of 12th graders and the CCSS may emphasize slightly different combinations of topics, the majority of the content is overlapping. For example, the NAEP Framework includes more in-depth knowledge of symmetry, and explicit expectations for estimation than the CCSS. The CCSS, on the other hand, expect students to work with polynomial functions and their attributes, while NAEP does not at this level. As such, students will be expected to have a stronger understanding of this content than NAEP expects. When the (+) standards from the CCSS and the NAEP objectives with a "*" are included, the two documents continue to have much in common. Regardless of whether they master mathematics at the college- and career-ready level of the CCSS or they continue to even more advanced content, students mastering the K-12 CCSS should be well prepared for the 12th grade NAEP exam.

In short, states that adopt the Common Core State Standards will prepare their students to do well on the 4th, 8th and 12th grade NAEP exams. Overall the CCSS and NAEP mathematics Framework describe expectations of comparable rigor for the end of 4th, 8th and 12th grades, with only minor differences.

Coherence and Focus

Because the NAEP served as an important resource for the CCSS, the CCSS and the NAEP Framework are similar in coherence and focus at the end of grades 4, 8 and 12. Both documents have a tightly defined set of content and skills at those grade levels, through which students build a strong foundational understanding of mathematics before beginning high school coursework and eventually college and careers. Both the CCSS and the NAEP Framework describe a substantially similar body of knowledge that students are expected to know by the end of grades 4, 8, and 12. Although the CCSS provide standards at each grade level, they reach similar thresholds by grade 4, 8 and 12 as the NAEP Framework.

Despite the similarities, there are differences with respect to coherence and focus. The Common Core State Standards provide more precision about the importance, progression and connections among topics by detailing expectations at each grade level in K-8. In contrast, and as a direct result of its design, the NAEP Framework describes content to be mastered by the ends of grades 4, 8, and 12 only. NAEP is designed as an assessment of the cumulative learning students have achieved by particular points in time, and because it assesses only at those times, it has not been necessary for the Framework to focus much attention on the learning that transpires prior to the assessment point. As a result, progressions are not as clear in the NAEP Framework although alignment at each benchmark grade is strong.

In short, while the Common Core State Standards and the NAEP Framework share some traits of focus, the CCSS serve as an important complement to the Framework by describing more precise and clear progressions at all grade levels.

1 The * denotes objectives that describe mathematics content beyond that typically taught in a standard three-year course of study (the equivalent of one year of geometry and two years of algebra). Therefore, these objectives will be selected less often than the others for inclusion on the assessment.

Conclusion

NAEP is a valuable, well-regarded resource. It has consistently defined high expectations for students and provided a common benchmark by which states can compare their achievement. Overall, the CCSS are well aligned to the NAEP Framework; there are areas, particularly with respect to focus and coherence where the NAEP Framework is less well aligned but that has more to do with the design and purpose of NAEP assessments than anything else. Policymakers can be confident that CCSS reflect the rigor of the current NAEP mathematics Framework and that in states where students master the CCSS in mathematics, that mastery should be reflected in 4th, 8th and 12th grade NAEP scores.

The CCSS also provide a valuable complement to NAEP. They define high standards in considerably greater detail than is necessary for the NAEP Framework, describing the detailed content progressions that support the attainment of those standards. Because the two documents are so well aligned overall, students who master the CCSS should be able to perform well on the NAEP, and potentially be even better prepared than they are today.

Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations. For more information about Achieve, visit www.achieve.org

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Appendix

Standards for Understanding Fractions, CCSS and NAEP Framework Compared

NAEP Grade 4	NAEP Grade 8	CCSS
G4.NPO.e) Connect model, number word, or number using various models and representa-tions for whole numbers, fractions, and decimals.	G8.NPO.b) Model or describe rational numbers or numeri-cal relationships using number lines and diagrams	CC.4.NBT.5 Use place value understanding and properties of operations to perform multi-digit arithmetic: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.
G4.NPO.a) Add and subtract: ▪ Fractions with like denominators	G8.NPO.d) Write or rename rational num-bers.	CC.2.G.3 Reason with shapes and their attributes: Partition cir-cles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
	G8.NPO.a) Perform computations with rational numbers.	CC.3.NF.1 Develop understanding of fractions as numbers: Un-derstand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
	G8.NPO.e) Interpret rational number operations and the relationships between them.	CC.3.NF.2 Develop understanding of fractions as numbers: Un-derstand a fraction as a number on the number line; represent fractions on a number line diagram.
		CC.3.NF.2a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partition-ing it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
		CC.3.NF.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
		CC.3.NF.3 Develop understanding of fractions as numbers: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
		CC.3.NF.3a Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3); explain why the fractions are equiva-lent, e.g., by using a visual fraction model.
		CC.3.NF.3b Express whole numbers as fractions, and recog-nize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

		CC.3.NF.3c Compare two fractions with the same numerator or the same denominator, by reasoning about their size; recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
		CC.3.G.2 Reason with shapes and their attributes: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part is 1/4 of the area of the shape.
		CC.4.NF.1 Extend understanding of fraction equivalence and ordering: Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size; use this principle to recognize and generate equivalent fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		CC.4.NF.2 Extend understanding of fraction equivalence and ordering: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2; recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		CC.4.NF.3 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers: Understand a fraction a/b with a > 1 as a sum of fractions 1/b. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		CC.4.NF.3a Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation (e.g., 3/8 = 1/8 + 1/8 + 1/8 and 3/8 = 1/8 + 2/8). Justify decompositions, e.g., by using a visual fraction model.
		CC.4.NF.3b Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

		CC.4.NF.3c Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
		CC.4.NF.4 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
		CC.4.NF.4a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
		CC.4.NF.4b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
		CC.4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example: If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
		CC.5.NF.1 Use equivalent fractions as a strategy to add and subtract fractions: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.) =
		CC.5.NF.2 Use equivalent fractions as a strategy to add and subtract fractions: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < 1/2.

		CC.5.NF.3 Apply and extend previous understandings of multiplication and division to multiply and divide fractions: Interpret a fraction as the result of dividing the numerator by the denominator (a/b = a ÷ b or a divided by b); solve word problems involving division of whole numbers leading to fractional answers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?