Comparing the Common Core State Standards in Mathematics to the Recommendations of the National Mathematics Advisory Panel

Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and the National Mathematics Advisory Panel's (NMAP) recommendations found in *Foundation for Success*.

Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career-ready requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering, and Mathematics (STEM) coursework in college.

Mathematics Content Recommendations by the National Mathematics Advisory Panel

The NMAP, a group of nationally-known mathematicians and experts on mathematics learning and education, was formed by executive order in 2006 to examine the "best available scientific evidence" regarding mathematics education and to make recommendations for its improvement. The Panel made a number of recommendations and identified "Benchmarks for the Critical Foundations" for progress in mathematics learning at each stage from pre-school through grade 7, with a particular focus on readiness for algebra and the major algebra topics all students should master. They also provided recommendations for the "The Major Topics of School Algebra," content which is normally found in high school coursework. As the NMAP recommendations do not fully describe content outside of that which is required to be prepared for and excel in algebra, a full comparison between the NMAP's recommendations and the CCSS is not possible. Yet given the evidence-based approach of the NMAP and their intense focus on the content and skills that build the foundation for success in algebra, there is great value in better understanding how the NMAP's recommendations compare to the Common Core State Standards.

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Achieve's Analysis

Achieve has analyzed the CCSS and NMAP's recommendations to determine how they compare in terms of **rigor, coherence, and focus**. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; they should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the CCSS to the NMAP's recommendations, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

Major Findings

- The CCSS and the NMAP's recommendations describe similar levels of rigor. Minor differences appear between the two documents in terms of when content is expected to be mastered.
- The CCSS are more coherent and focused than the NMAP's recommendations. The CCSS emphasize similar content, but provide clearer and more precise content expectations at each grade level and progressions of learning across the grades.

Detailed Findings

Rigor

Achieve's analysis indicates that overall the documents are similarly rigorous, and describe substantially similar bodies of knowledge, though there are some noteworthy differences between the CCSS and the NMAP's recommendations.

- **Elementary grades:** In general, the CCSS and the NMAP recommend programs of comparable rigor through grade 4. Both expect students at the end of that grade to be able to add and subtract with whole numbers, and to understand place value and the meaning and uses of fractions. As a result, both documents lay out a similar foundation in the early grades.
- **Middle grades:** There are also substantial similarities in the middle grades. Both expect students at the end of grade 8 to be proficient in all operations involving positive and negative integers, fractions, and decimals. They both also address the need for students to be able to solve problems involving percentages, ratios, and rates, as well as proportionality. The CCSS and NMAP also treat geometry and measurement in very similar ways, both expecting students to be able to analyze the properties of and solve problems involving a variety of shapes. While the CCSS more fully describe this content, both documents require the mastery of content needed for advanced mathematics in high school. A few differences exist as well, primarily in regards to sequencing. Some NMAP recommendations are aimed at earlier grades, but in most cases the difference is only one year from what the CCSS require. For example, the NMAP recommendations introduce the relationship between similar triangles and the slope of a line by the end of grade 7; the CCSS include this material in benchmarks for grade 8, along with complementary expectations about lines. By introducing this critical content at the same time, the CCSS create an environment where the content is mutually reinforcing and will likely lead to a deeper understanding of lines and linear equations on the part of students.

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- **High school:** The comparison between those CCSS's required of all students to be college-and career-ready and the NMAP recommendations for algebra shows significant similarity. Both require students to work with symbols and expression linear equations; quadratic equations; and a variety of functions including linear, quadratic, exponential, and trigonometric functions. As a result, by the time students have mastered the knowledge and skills necessary for success in entry-level, credit-bearing courses and 21st century careers in the CCSS, they will also have addressed the vast majority of content found in the NMAP's recommendations. Although the CCSS and the NMAP recommendations include largely the same algebra-related content, the CCSS treat some of the content as expectations only for students intending to pursue further study of mathematics and science, while the NMAP considers all its recommended algebra content to be essential for all students. Even so, the NMAP recommendations include only two topics, permutations and combinations and the Fundamental Theorem of Algebra, for all students, which the CCSS designate with a (+).

In short, the CCSS and the NMAP recommendations are very well aligned. The few differences between the CCSS and the NMAP recommendations for grades K-7 relate to the time at which students are expected to master particular content. Most importantly, though, nearly all of the K-7 NMAP recommendations are included in the K-7 progression of the CCSS.

Coherence and Focus

The NMAP's recommendations for grades K-7 emphasize the concepts and skills that are the foundation for success in algebra: number, geometry, and measurement. The CCSS also treat preparation for algebra as a critical goal for K-7 mathematics, and describe a very similar development of content for those grades. With respect to the Panel's recommendations for high school, Achieve found that the CCSS include the majority of the goals for algebra itself in their expectations for all students' high school mathematics learning. As such, policymakers can be assured that students who complete the content progressions in the CCSS will have increased their mathematical content knowledge in a fashion similar to the progression recommended by the NMAP.

There are a few specific differences between the two documents that result in the CCSS providing greater utility to teachers.

- The CCSS explicitly describe the skills students are expected to know and concepts they are expected to understand for each grade level and usually establish links between understanding and skills. The NMAP recommendations focus on outlining the content that is necessary preparation for algebra and in the study of algebra content itself by the end of certain grade levels. Thus, the CCSS provide more specific guidance to educators in terms of grade-by-grade instruction.
- The CCSS more completely describe the content students should learn across all domains of mathematics. As a result of its purposes, the NMAP recommendations do not fully describe content outside of that which is required for algebra, thus it is of little use when considering the breadth of mathematics content students must learn to be college- and career-ready such as geometry, probability and statistics, and modeling. The CCSS, however, clearly describe the expectations across all of these critical areas of learning, making it a more useful document to educators.
- The CCSS describe content progressions across grades as well as the expectations themselves in each grade. In contrast, and as a direct result of its design, the NMAP recommendations describe content to be mastered by the ends of key grades, such as "By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers." As a result, progressions are not as clear as they are in the CCSS. By organizing content in strands that span several grades at a time, the CCSS more clearly describe how content is developed over time.

In short, while the CCSS and the NMAP recommendations share some traits of coherence and focus, the CCSS provide more precise and clear progressions, more complete descriptions of content across the discipline, and identify content students should learn at each grade, thereby providing greater support to teachers and students alike.

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Conclusion

Overall, the CCSS and the NMAP recommendations are similarly rigorous. The two documents describe a substantially similar body of knowledge associated with algebra. Policymakers can be assured that in adopting the CCSS they will be setting learning expectations for students that are similar to those advocated by the NMAP.

Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations. For more information about Achieve, visit www.achieve.org

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