

# Comparing the Common Core State Standards in Mathematics to California and Massachusetts Standards

## Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and the standards of two states that have been leaders in mathematics education: California and Massachusetts.

## Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra, and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7<sup>th</sup> grade and mastered the content and skills through the 7<sup>th</sup> grade will be well prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career-ready requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering, and Mathematics (STEM) coursework in college.

## Mathematics Standards in California and Massachusetts

**California's** well-regarded Mathematics Content Standards (California standards) articulate expectations for students in kindergarten through grade 7 that are organized in five strands: number sense; algebra and functions; measurement and geometry; statistics, data analysis, and probability; and mathematical reasoning. Performance standards also are presented using sample problems and activities. The mathematics studied in grades eight through twelve falls naturally under the discipline headings Algebra I, Geometry, Algebra II, and so forth. Apart from expecting that all students begin Algebra I in grade 8, the California standards do not specify grade-level student expectations above grade 7.

California's standards have received well-deserved recognition for their quality and rigor. However, while the standards themselves are rigorous, laying out a pathway through Calculus, students are not actually required to learn all of the content in the standards. California state policy requires only two years of mathematics, including Algebra I, for graduation. Thus a student in California may graduate having mastered little of the mathematics in the California standards.

**Massachusetts** has attracted national attention for its education reforms and for the progress the state has made in closing achievement gaps and boosting student achievement overall. Massachusetts students are regularly among the nation's top performers on NAEP; and in 2009, Massachusetts scored second in the world on the Trends in International Mathematics and Science Study (TIMSS). Steady gains on the state assessment (MCAS) have burnished the education system's reputation.

The Massachusetts Curriculum Framework (Massachusetts standards) is widely regarded as one of the best sets of state standards. It is organized around five mathematical strands: number sense and operations; patterns, relations, and algebra; geometry; measurement; and data analysis, statistics, and probability. The framework also includes standards for Algebra I, Geometry, Algebra II, and Precalculus. Performance standards are developed using sample problems and activities.

However, Massachusetts does not explicitly require its students to successfully complete any courses for graduation, choosing instead to leave graduation course requirement decisions to districts. Therefore, while Massachusetts has rigorous standards that set the stage for strong performance by students on major assessments, there is no explicit requirement that students master them.

## Achieve's Analysis

Achieve analyzed the CCSS; the California standards for K-7, Algebra I, Geometry, Algebra II, and Probability and Statistics; and the Massachusetts standards for K-8, Algebra I, Geometry, Algebra II, and Precalculus to determine how they compare in terms of **rigor, coherence, and focus**. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics, and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; the standards should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the three sets of standards, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

## Major Findings

- The CCSS are similarly rigorous to the California and Massachusetts standards. While all three describe similar content, the CCSS go beyond both sets of standards by identifying the level of content required of all students to graduate from high school college and career ready.
- The CCSS are more coherent than the California and Massachusetts standards. The CCSS emphasize similar amounts of content in each grade level, but provide clearer and more precise progressions of learning across the grades.
- While there are a number of similarities between the CCSS and the California and Massachusetts standards, there are several key differences in coherence and focus which set the CCSS apart as a better set of standards.

## Detailed Findings

### Rigor

Both Massachusetts and California have highly-regarded, rigorous mathematics standards. Achieve has used both for years as benchmarks against which to compare other state standards. The Massachusetts standards are widely credited as the foundation for the state's steady improvement and internationally competitive performance. Because leaders in both states have made clear that they will not adopt the CCSS if they are less rigorous than current state standards, the rigor of the math standards is a topic of considerable interest and debate in each.

Achieve's analysis indicates that overall the three sets of standards are similarly rigorous, and describe substantially similar bodies of knowledge, though there are some noteworthy differences between the CCSS and these particular state standards.

✓ **Elementary grades:** Through grade 4, the CCSS and the California and Massachusetts standards are generally comparable in rigor. By the end of that grade students are expected to understand the four basic operations with whole numbers, place value, and the meaning and uses of fractions. Consistent with the standards of many high-performing countries, data, probability and statistics content is deemphasized in the early grades, constrained primarily to the creation of graphs to represent data. The CCSS begin more formal work with data in Grade 6, while Massachusetts and California do so in 2<sup>nd</sup> grade. However, the strong foundation in number sense provided by the CCSS in the early grades will allow students to progress quickly in middle and high school through data, probability and statistics.

✓ **Middle grades:** There also are substantial similarities in the middle grades. For example, all three documents include some of the major hallmarks of algebra—proportionality, linear expressions and inequalities, and using equations and inequalities to solve real life and mathematical problems—which prepare students well for more advanced mathematics in high school. In 8<sup>th</sup> grade specifically, the CCSS include a significant amount of Algebra I content, but the full coverage of Algebra I is treated in high school. However, students who meet the standards at the end of 7<sup>th</sup> grade would be prepared for Algebra I. In fact, the CCSS's approach to the early grades does a better job preparing students for algebra in 8<sup>th</sup> grade than either the California or Massachusetts standards.

✓ **High school:** Overall the content of the secondary level math standards is quite similar across all three documents. For example, all three include linear, quadratic, and exponential expressions; equations; and functions in the expectations for high school students. These topics are important preparation for the mathematics found in later courses, including entry-level, credit-bearing coursework in college, such as College Algebra. The biggest differences between the documents arise because the California math standards include math through Calculus (although Calculus is not required of students), while Massachusetts and the CCSS include content through Pre-calculus. Additionally, the CCSS pay greater attention to creating and using mathematical models based on real world contexts than do either Massachusetts or California. This knowledge is highly valued by employers in many sectors, including those that hire employees with little or no formal postsecondary education, making its prominence in the CCSS an important difference from both the Massachusetts and California standards.

*More about Algebra I: Algebra is the gateway for high school mathematics and preparation for postsecondary education. California expects all students to take Algebra I in 8<sup>th</sup> grade, while Massachusetts does not.*

While the documents describe similar content, they nonetheless communicate different levels of rigor. The CCSS specifically define the knowledge and skills necessary for success in entry-level, credit-bearing courses and 21<sup>st</sup> century careers. Neither Massachusetts nor California similarly identifies college- and career-ready standards. While the CCSS Initiative appropriately steered clear of defining course-taking requirements for high school graduation, to




meet the college- and career-ready standards all students would have to take three years of math through Algebra II, or the equivalent.<sup>1</sup> Similarly, Achieve has urged states to require all students to complete a curriculum aligned with college- and career-ready standards, including math through Algebra II. Twenty-one states now require students to complete such a course of study. In contrast, California requires students to take only two years of math including Algebra I. Massachusetts delegates decisions about course-taking requirements to local school districts.

While symbolically and politically important, the debate in each state about the relative rigor of the high school standards seems to miss the larger educational point – very rigorous standards that students are neither required to meet nor have the opportunity to learn are not nearly as valuable as required standards that prepare all students for postsecondary success.

## **Coherence and Focus**

Achieve's analysis indicates that the Common Core State Standards, California's, and Massachusetts' standards share some key traits of coherence and focus. For example, all three documents develop an understanding of adding, subtracting, multiplying and dividing whole numbers over an average of about three years. They also all emphasize a few key topics and develop connections among those topics at each grade level, which helps to ensure a greater depth of learning. All three address connections across the number, algebra, geometry, and data strands, and use tools such as number lines and graphic representations to illustrate these links. Mathematical reasoning is addressed in all three documents and all also highlight the importance of explaining and justifying solutions to problems.

Despite these similarities in coherence and focus, there are critical differences between the CCSS and the California and Massachusetts standards. The result of these differences is that the Common Core State Standards provide more precision about the importance, progression, and underlying concepts of content across the grades.

-  The Common Core State Standards provide a more coherent progression of several major topics in mathematics. For example, with respect to multiplication and division with fractions, the CCSS lay out a progression from grades 4-7 where there is a noticeable and reasonable increase in rigor from one grade to the next. In contrast, the California standards attend to this content in a shorter span, from grades 5-7. In Massachusetts, the progression of this topic goes from grades 5-8, but there is little or no progression from grades 5-6 and from grades 7-8. As a result, students who master the CCSS in this area will have a stronger foundation and deeper understanding of multiplication and division with fractions, which will more effectively support them in their later work in algebra.
-  Further, the CCSS provide a precise definition of the core concepts and skills students must master in grades 5-7 to be well prepared for algebra (e.g., key aspects of rational numbers and geometry), and a very clear grade-by-grade progression of topics in each area. In contrast the California standards in particular are significantly less precise, and neither Massachusetts nor California provide a sufficiently clear progression across the grades. Therefore, students in each state must make a more abrupt transition from the concrete skills learned through 4<sup>th</sup> grade to the more abstract reasoning required for algebra, rendering them less well prepared than students who master the CCSS would be.
-  Like the standards of many high-performing countries including Hong Kong and Japan, the Common Core State Standards focus on the properties of operations<sup>2</sup> as a way of gradually preparing students for algebraic thinking. Both the California and Massachusetts standards introduce algebra through the creation, description, extension, and explanation of patterns, drawing heavily on the work of the National Council of Teachers of Mathematics. The CCSS include this content in Grades 4 and 5 after sufficient focus has been provided to the core knowledge associated with numbers and operations in Grades 1-3.

<sup>1</sup> The CCSS organize the content into strands, not into courses. In response to requests from many states, Achieve is leading an effort to organize the standards into model courses and curricular pathways. There will be a traditional course sequence that includes Algebra I, Geometry, Algebra II and culminates in one of several 4<sup>th</sup> year courses, including but not limited to Pre-calculus, AP Statistics and a course that emphasizes modeling. There will also be an integrated math sequence that includes the identical content but combines algebra and geometry into several courses, much as many high performing countries do. Though these course sequences start in 9<sup>th</sup> grade, Achieve will also produce a version of each that starts Algebra I in 8<sup>th</sup> grade. These model pathways will be included as an appendix to the CCSS.

<sup>2</sup> Associativity and commutativity of addition and multiplication, distributivity of multiplication over addition, the additive identity property of 0, and the multiplicative identity property of 1.

- ✓ The Common Core State Standards more clearly define the important and necessary conceptual knowledge for building connections and deepening understanding. The relevance of particular conceptual knowledge may potentially be inferred from the content of individual topics in the California and Massachusetts standards, but is not explicitly specified. This is particularly evident in how the three documents treat number concepts in grades K-3. Here, the California and Massachusetts standards focus on the raw skills associated with sums and differences, but do not explicitly describe their relationships to the underlying concepts of place value. The result is that students who master the CCSS in these grades will likely have a better foundation for work with more advanced mathematics in later grades.

In short, while the Common Core State Standards and the California and Massachusetts standards share some traits of coherence and focus, the CCSS ultimately improve on both of the states' standards. The CCSS provide more precise and clear progressions, greater emphasis on conceptual knowledge, and a smoother development of algebraic reasoning.

## Conclusion

Overall, the Common Core State Standards, the California standards, and the Massachusetts standards describe similar bodies of knowledge. All three make clear that curricula should be focused on a tightly defined set of content and skills each year in order for students to build a strong foundational understanding of mathematics, with the CCSS giving more emphasis than the others in the early grades on number concepts and skills. Policymakers can be assured that in adopting the Common Core State Standards, they will be setting learning expectations for students that improve upon those currently set by California and Massachusetts, and that the college- and career-ready bar in the Common Core State Standards is set at a level that currently surpasses the level of the mathematics required for graduation in both states.

*Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations. For more information about Achieve, visit [www.achieve.org](http://www.achieve.org)*