

Lesson/Unit Name: Unit Rate and Constant of Proportionality

Content Area: Mathematics

Grade Level: 7

Reviewer 1

Overall Rating:

E

Exemplar

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
- ✓ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
- ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

This review contains feedback for the four lessons (7-10) in Topic B of the Ratios and Proportional Relationships module for Grade 7. In the overview of the topic, content standards 7.RP.2b, 7.RP.2c, and 7.RP.2d are listed as the focus standards for these lessons. It was found that the lessons address the overall cluster expectation that "Students analyze proportional relationships and use them to solve real-world and mathematical problems." and the overarching expectation for 7.RP.2 to "Recognize and represent proportional relationships between quantities." to the depth expected for these three standards.

Standards for Mathematical Practice 1, 2, and 4 are listed as the focus practice standards for the topic. It is appropriate for these standards to be listed for the content in Topic B. Throughout these four lessons students are asked to make sense of real-world problems dealing with proportional contexts and engage in productive struggle to solve these problems (SMP1). As they work with these problems, students are asked to take the quantities from the problem contexts, work with those quantities mathematically, and then put their solutions back into the context to see if they make sense (SMP2). Students engage in SMP4 when they are asked to represent proportional relationships using mathematical models/equations. SMP4 also asks students to analyze multiple representations, such as tables, diagrams, and graphs, and to draw conclusions regarding the mathematical patterns inherent in those representations. This module definitely asks students to do just that as they draw conclusions about and write equations to describe the proportional relationships in the various representations.

There is a good balance of procedures and conceptual understanding throughout Topic B. A plenitude of tables, equations, graphs, and verbal descriptions were all used to illustrate the meaning of proportional relationships within real-world problem contexts.

The only remaining suggestion is related to the additional conceptual understanding of proportional relationships as a relationship between two quantities to be gained from the use of a few more diagrams. 7.RP.2b states that representations should include diagrams, such as tape diagrams and double number lines, such as those on page 67 (maybe should be page 69) and on page 84 (maybe should be page 86). (Note: I think you forgot to re-paginate after adding the graphics on page 67 for the deer population problem.)

With regards to diagrams, page 4 of the 6-7 Ratio and Proportional Relationships progression states:

- "Because ratios and rates are different and rates will often be written using fraction notation in high school, ratio notation should be distinct from fraction notation. Together with tables, students can also use tape diagrams and double number line diagrams to represent collections of equivalent

ratios. Both types of diagrams visually depict the relative sizes of the quantities."

- "Tape diagrams are best used when the two quantities have the same units. They can be used to solve problems and also to highlight the multiplicative relationship between the quantities." Perhaps you are planning to model the use of tape diagrams and a few more double number lines to represent proportional relationships in Topics A and C.
- "Double number line diagrams are best used when the quantities have different units (otherwise the two diagrams will use different length units to represent the same amount). Double number line diagrams can help make visible that there are many, even infinitely many, pairs in the same ratio, including those with rational number entries. As in tables, unit rates appear paired with 1." (Also see the illustrations of diagrams through the entire progression.)

Overall, the four lessons in Topic B of the Ratios and Proportional Relationships module meet most of the criteria in this dimension.

Rating: **3 – Meets most to all of the criteria in the dimension**

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- ✓ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- ✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- ✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
 - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
 - **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures

In 7th grade, all of the Ratio and Proportional Relationships standards are major work of the grade and the lessons in Topic B of the module are targeted, succinct, and to the depth of understanding expected for this major work. The topic is handled in a grade-appropriate manner and does not expect students to master content beyond the target 7th grade expectations for these lessons.

Topic B of the Ratios and Proportional Reasoning module shows consideration of the coherence with the foundational knowledge and skills from 6.RP.1, 6.RP.2, and 6.RP.3 with regards to unit rate and solving unit rate problems by activating the prior knowledge from those standards in Lesson 7. The overview speaks to making connections in future lessons and modules to scale factor, scale drawings, and percent. You also may want to consider connecting the major work of proportional reasoning in the overview to the module that addresses supporting clusters 7.SP.A which asks students to "Use random sampling to draw inferences." and cluster 7.SP.C which asks students to "Investigate chance processes and develop, use, and evaluate probability models." Using proportional reasoning to draw inferences and develop theoretical probability models as described are a good connection to ratio and proportional relationships.

There is evidence in Topic B of the Ratio and Proportional Relationships module to the common core shift of Rigor. Conceptual understanding of proportional relationships is developed through thoughtful contextual problems that ask students to use tables, verbal descriptions, graphs, and equations to describe proportional relationships. You may want to consider some additional diagrams, such as double-tape diagrams and double number lines as shown throughout the grades 6-7 Ratios and Proportional Reasoning learning progression. These visual diagrams will help to further student conceptual understanding of a proportional relationship as the relationship between two quantities and understanding of the unit rate as "per one" and the meaning of $(1, r)$ as expected in 7.RP.2d. Procedural skill is also evident throughout Topic B as students are asked to write and solve equations in the form of $y = kx$ and to find the constant of proportionality from different representations. Lastly, the plethora of real-world problems enable students to apply their understanding of the constant of proportionality and

(when called for in the standards for the grade) to be performed quickly and accurately.	<p>proportional reasoning in different contexts. It is clear in the overview that opportunities for more application will be provided in the future in the context of scale drawings and the study of percents.</p> <p>Overall, Topic B of the Ratios and Proportional Relationships module meets all of the criteria in this dimension.</p>
<p>Rating: 3 – Meets most to all of the criteria in the dimension</p>	

Dimension III – Instructional Supports

<p><i>The lesson/unit is responsive to varied student learning needs:</i></p> <ul style="list-style-type: none"> ✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media. ☐ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline. ✓ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking. ✓ Addresses instructional expectations and is easy to understand and use. ✓ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners. <ul style="list-style-type: none"> – Supports diverse cultural and linguistic backgrounds, interests and styles. – Provides extra supports for students working below grade level. – Provides extensions for students with high interest or working above grade level. <p><u><i>A unit or longer lesson should:</i></u></p> <ul style="list-style-type: none"> ✓ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share). ✓ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently. ✓ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ✓ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately. 	<p>Topic B of the Ratios and Proportional Relationships module includes clear guidance for the students in the form of stated student outcomes at the beginning of each lesson. In addition to the mention of using graphing calculators, there are many free on-line applications for consideration that would be appropriate for student exposure to technology to access the concepts in the module. Graphing calculators and on-line applications are appropriate technology tools, in addition to pencil and paper for students to learn to use to generate linear graphs.</p> <p>The topic does address the instructional expectations and provides guidance for the teacher. The lessons follow the same format throughout, are clearly written, and provide guidance for the teacher. Solutions are provided for the teacher for the lesson problems, problem sets, and exit tickets. However it is not clear how the problem sets and/or exit tickets should be used to gauge student learning. What is a teacher to do when a student does not achieve the mastery expected in either the exit ticket or the problem set? What answers would indicate possible misconceptions?</p> <p>There are many opportunities for students to be precise and accurate with mathematics language and representations throughout the topic.</p> <ul style="list-style-type: none"> - Topic B specifically addresses the meaning of the constant of proportionality throughout the lessons as being the same as the unit rate (p. 62). Teachers need to realize is only true when the constant of proportionality is specifically stated in terms of "per one...". Also, page 4 of the Ratios and Proportional Relationships learning progression states "As in tables, unit rates appear paired with 1." For example, on page 73, in both #2 and #4 the constant of proportionality is stated in terms other than "per one". In #2, Li runs 2 laps in 5 minutes. Is it clear to the students that this is $\frac{2}{5}$ of a lap or 0.4 of a lap per minute? Likewise for #4 do students really understand that 6 packages for \$5 means that you buy 1.2 or $\frac{6}{5}$ packages per dollar? - Perhaps the solution to the above challenge is a matter of simply providing written unit labels for all the solutions to lesson problems, exit tickets, and problem sets. Also consider suggesting to teachers that students be required to write or express the constant of proportionality not just in the numerical form of the ratio of y/x, but also in words such as "The unit rate or the constant of proportionality for this situation is $\frac{6}{5}$ packages per dollar." It is common for students to lose track of the units and their meaning in the context of the situations, especially when dealing with ratios. This is why it is so important for students to not just give numerical responses, but to frame their responses using verbal descriptions of the context and using unit labels. The practice of answering in complete sentences and with unit labels also supports the precision of language expected in SMP6. - The emphasis on "per one..." in the understanding of the constant of proportionality and unit rate also directly connects to the graphical meaning of the unit rate as defined by "r" in the point (1,r) in 7.RP.2d. In grade 8, this understanding of the meaning of (1,r) in the graph of a proportion will build
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to the understanding of $(1,r)$ where "r" is the Unit Rate is the same as the rate of change or the slope.

- Lastly, remind students that ratios can also be represented with a colon. E.G. There are 2 boys for every 3 girls which is the same as 2 boys/3 girls or 2 boys:3 girls.

The problems in Topic B are thought provoking, set in real world context, and provide opportunities to elicit mathematical thinking. They are of high quality and are aligned to the goals for the topic. As mentioned earlier, reflect on whether at the pace suggested for the lessons, whether or not students have enough time in the lessons to actually "think" about the problems. These RP standards are major work of Grade 7 and you may want to strongly consider if four days is enough time for the work on 7.RP.b-d as described in Topic B. These 7.RP.b-d standards are a critical foundation for "rate of change" and the concept of slope for the linear function work in Grade 8. You may want to also reflect on the amount of work expected for the Exit Tickets. For example, a time of 5 minutes is suggested for the Exit Ticket that spans pages 70-71. The expectations of the ticket include written explanations and justifications in addition to procedural skill. The problem is very good, but is there enough time for students to thoughtfully engage in the mathematics?

Multiple representations including tables, graphs, equations, verbal contexts, and other visuals help to provide support for students with different learning needs, including ELLs. The inclusion of the Frayer Model (p. 61) and scaffolding suggestion boxes on the right-side of multiple pages throughout the lessons facilitate a mix of instructional approaches and supports for students needing additional support. It is admirable that the scaffolding boxes in addition to supports for struggling students, also include in some instances, opportunities for extensions of the content for students ready to move forward during the lesson.

Overall, Topic B of the Ratios and Proportional Relationships module meets many of the criteria in this dimension.

Rating: 2 – Meets many of the criteria in the dimension

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- ✓ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- ✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:

- ✓ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

Throughout the module, students are sequentially supported to show independent mastery of the standards. The exit tickets, as do the questions and problems posed throughout the lessons, provide opportunities for teachers to gauge the level of student understanding of the standards, and support students as needed. The questions use grade-level language in student prompts, and answer keys are provided to provide guidance for interpreting student performance. It may be helpful to provide more guidance for teachers on how to identify and address student error patterns and misconceptions that may arise during the lessons, exit tickets, and problem sets. This addition would strengthen formative assessment practices.

A mid-module assessment provides additional support for teachers to gauge student mastery of standards on a more formal level than during the lessons. This mid-module assessment is meant to assess student understanding for Topics A and B which includes standards 7.RP.2a-d. A rubric, on a scale of 1-4 with criteria for each level of mastery, is provided. You may want to reflect on the level of expectation for the solutions against the depth of expectations of the standards. For example, the solution for 3a

is presented simply as $1/5 = k$. What does this ratio mean in relation to the context of the problem? Students should be expected to know that the pop singer earns \$1 for every \$5 of sales by the music store, in addition to just the ratio for the constant of proportionality. Once again, are unit labels warranted here? This reviewer wonders how many students would be able to accurately state the relationship between the two quantities as a version of "For every \$5 in sales, the singer earns \$1."

There are no specific pre-assessment or student self-assessment activities included in Topic B. You may want to consider these as refinements to the module.

Overall, Topic B of the Ratios and Proportional Relationships module meets many of the criteria in this dimension.

Rating: 3 – Meets most to all of the criteria in the dimension

Summary Comments

Overall this is a strongly constructed topic that is exemplary for teaching and learning. The lessons address major work of Grade 7 and capture both the letter and the spirit of the Ratios and Proportional Reasoning standards for the grade.

The only suggested refinements at this time are:

- using double-tape diagrams to model and represent the relationship between two quantities with like units in proportional relationships.
- adding more time to these lessons because they include major work of the grade and conceptual understanding of these standards is foundational to the linear function work in Grade 8.
- investigating opportunities for more technology integration, such as on-line graphing applications.
- expecting that students be able to describe the constant of proportionality numerical value (k) as the unit rate with accurate unit labels that are related to the context as a "per one" unit in every situation throughout the module. Guidance should be provided for the teacher regarding this expectation.
- adding teacher support and guidance for how to identify and address common errors or misconceptions that surface during lessons, problem sets, exit tickets, or in assessments.

Reviewer 2

Dimension I – Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
- ✓ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
- ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

This unit for Unit Rate and Constant of Proportionality consists of 4 lessons. Content standards addressed within this unit address three parts of 7.RP.2 - Recognize and represent proportional relationships between quantities and 7.G.A.1 - Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. The problems and activities used in the lesson support these content standards. There is some concern that addressing four standards within 4 days might not allow for the full depth of understanding to be developed but it is recognized that these four lessons occur within a broader unit where these standards might be reinforced.

The Standards for Mathematical Practice addressed in this lesson are MP.1-Make sense of problems and persevere in solving them, MP.2-Reason abstractly and quantitatively, and MP.4-Model with mathematics. Places within the lesson where these Mathematical Practices might be emphasized are marked to draw the attention of the teacher. A brief overview of how these three Mathematical Practices are embedded within the lessons is

provided on pages 5-6. This will be very useful in helping teachers emphasize these Mathematical Practices within their teaching.

By presenting situations that students can connect to real-life experiences (such as baking cookies or the amount of gasoline used by a car), the unit provides a strong connection between mathematical procedures and a deeper conceptual understanding. Different representations, such as equations, tables, and graphs, are used to model the problem situations presented to students. The emphasis of the instruction focuses on equations and it might be beneficial to provide more opportunities to connect the different representations.

Rating: 3 – Meets most to all of the criteria in the dimension

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- ✓ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- ✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- ✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
 - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
 - **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Focus: One of the major works for grade 7 CCSS is to develop understanding of and apply proportional relationships. The lessons included in this unit provide students many opportunities to work with proportional relationships using a variety of representations (tables, graphs, equations, and word problems). The lessons provide emphasis for:

- distinguishing proportional relationships from other relationships
- graphing proportional relationships and understanding the unit rate informally as a measure of the steepness of the related line.

Coherence: To begin this unit, students would need to be confident with the grade 6 CCSS standards for ratio and proportion. The work of this unit in grade 7 will set the stage for future learning with the grade 8 CCSS Expressions and Equations standards. Opportunities for connections addressed in the lessons include 7.G.A.1 to compute actual lengths for scale drawings and reproduce a drawing at a different scale.

Rigor: The problems presented to students are framed in situations that should be familiar to most students, such as baking cookies in Lesson 7 or determining the amount of money earned in Lesson 8. The lessons provide several opportunities to emphasize the connections between the variety of representations to help students develop a deeper conceptual understanding. Several of the tasks have the potential to develop conceptual understanding - there is a need for caution that the teacher not tell the students too much so there is a balance between the conceptual understanding and procedural skill.

Rating: 3 – Meets most to all of the criteria in the dimension

Dimension III – Instructional Supports

The lesson/unit is responsive to varied student learning needs:

- ✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
- ✓ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
- ☐ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
- ✓ Addresses instructional expectations and is easy to understand and use.
- ✓ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
 - Supports diverse cultural and linguistic backgrounds, interests and styles.
 - Provides extra supports for students working below grade level.
 - Provides extensions for students with high interest or working above grade level.

A unit or longer lesson should:

- ✓ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
- ✓ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
- ✓ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- ☐ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Several tools are included within the unit to support teachers in their implementation of this unit:

- The overview on page 3 provides a broad description of the unit to provide teachers with a "big picture" view of the expected student learning.
- The "Focus Standards for Mathematical Practice" will help teachers understand what these practices should look like within the unit. It is also helpful that places where these practices should be evident are noted by "MP1", "MP2" or "MP4" and highlighted within the lessons.
- The "Terminology" section on page 6 will be useful for teachers in using correct academic vocabulary and the "Suggested Tools and Representations" on page 7 provide a summary of the representations that will be used within the unit. The terms "constant" and "variable" is discussed on page 61 but not listed in the "Terminology" section on page 6. You might also consider adding "dependent variable" and "independent variable" to the vocabulary list for students. The option for students to use a Frayer model to define terminology is a helpful suggestion to differentiate vocabulary instruction for learners. The unit makes a clear distinction that a key understanding of "constant of proportionality" is connected to the previously learned "unit rate".
- Including samples of student responses for discussion questions and opportunities for scaffolding/extensions (as seen on pages 59-60 of Lesson 7) will be helpful for teachers new to the profession or mathematical content.
- Supports such as "Points to Remember" and "Lesson Summary" are useful in keeping the "big picture" in mind.

Each lesson is presented in an organized and consistent framework that is easy to follow. Exit tickets are provided so teachers have the opportunity to uncover potential student misunderstandings and address these the following day. More detail about how teachers might adjust the following day's instruction would be beneficial.

There is an error on page 78 of Lesson 9. Al's produce stand is listed that 7 ears of corn cost \$1.50. Using $y=0.21x$, the cost should be \$1.47. Also, Barbara's 13 ears of corn should cost \$2.86 using $y=0.22x$.

The lessons appear to be teacher-directed and rely on the teacher to initiate conversations. While the problems are engaging, the pace might not allow enough opportunity for students to engage in productive struggle. Although there is some time allowed within the lessons (for example, in Lesson 8 there are two problems completed within a 33 minute time frame), the problems are rather narrow and appear that they will be solved in a particular way. For example in the first problem in Lesson 8 about having enough gasoline to make it to the gas station, students are directed to find the constant of proportionality and write an equation. Consider including an opportunity within the four days for students to struggle with a task that does not have an immediate solution path.

The lesson describes a gradual release of responsibility with Lesson 7 opening with a whole class discussion of a problem and Lesson 10 opens with a brief review and then students working individually on problems.

The lessons provide multiple opportunities for differentiation, such as the use of a Frayer model for vocabulary terms in Lesson 7 and using a kinesthetic approach to open Lesson 8.

Rating: 2 – Meets many of the criteria in the dimension

Dimension IV – Assessment

<p><i>The lesson/unit regularly assesses whether students are mastering standards-based content and skills:</i></p> <ul style="list-style-type: none"> ✓ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. <p><u><i>A unit or longer lesson should:</i></u></p> <ul style="list-style-type: none"> ✓ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures. 	<p>There are varied opportunities for assessment within this unit and an overview is provided on page 7. The teacher can observe students during classwork and class discussions. Students complete an exit ticket at the end of each lesson and sample responses are provided for teachers. The unit also contains a MidModule Assessment Task to evaluate student learning. A rubric and answer keys are provided to provide consistency in grading of these assessments.</p> <p>Assessments are written in language that should be accessible to typically developing students. Consideration might be given to providing commentary about how these assessments might be used with struggling readers or ESL students since so much reading is required to solve the problems.</p> <p>Constructed response items allow students to describe their reasoning. The assessments are often directive in what type of representation should be used. For example, on the MidModule Assessment Task, the first problem requires the student to use a table and the second problem requires the student to use a graph on a coordinate plane. Perhaps an open-ended item where students solve the problem and explain their reasoning with a representation (or multiple representations) they choose could strengthen the assessment.</p>
<p>Rating: 3 – Meets most to all of the criteria in the dimension</p>	

Summary Comments

Overall, this is a very well-constructed unit with four lessons that should be easy for teachers to implement in the classroom. The content aligns strongly with the CCSS for grade 7 ratio and proportionality. There is a slight error in the calculations on page 78 of Lesson 9 that should be corrected. Including opportunities for differentiation, such as the use of Frayer models and kinesthetic learning, provide teachers with suggestions to meet the needs of their learners. There are multiple options for assessment including formative and summative assessments with answer keys and rubrics provided.

Reviewer 3

Dimension I – Alignment to the Depth of the CCSS

<p><i>The lesson/unit aligns with the letter and spirit of the CCSS:</i></p> <ul style="list-style-type: none"> ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ✓ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS. 	<p>The identified standards are major standards of focus that are critical to the implementation of the CCSSM.</p> <p>The in-depth strategies and scaffolding for the lessons are designed to ensure student understanding of the content.</p> <p>If all lessons are allowed to develop completely, then the essence of the identified standards will be met.</p> <p>The full description of the Focus Standards for Mathematical Practice on pages 5 and 6 describes how the standards are applied in the unit. The identified Standards for Mathematical Practice MP.1 (Make sense of problems and persevere in solving them.), MP.2 (Reason abstractly and quantitatively.) and MP.4 (Model with mathematics.) are embedded in a grade-appropriate manner and are connected to the content that is being</p>
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addressed. MP.1 is addressed at the beginning of each lesson as students are asked to struggle with the problem in a more inquiry-based approach. MP.2 is addressed several times as students make sense of the problem and reason abstractly. MP.4 is evident as students model real-world situations.

There is a balance of mathematical procedures and deeper conceptual understanding. Conceptual understanding was addressed through making connections and the use of multiple representations.

The class discussion components are good examples of driving discussion to lead to the conclusion of the content.

Rating: 3 – Meets most to all of the criteria in the dimension

Dimension II – Key Shifts the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- ✓ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- ✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- ✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
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 - **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

FOCUS: As described in the overview and the Focus Standards, this unit has a specific focus, targeting the student learning that the unit rate of a collection of equivalent ratios is called the constant of proportionality and can be used to represent proportional relationships with equations of the form $y=kx$, where k is the constant of proportionality. Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation. All of the above are part of the Major work for seventh grade in Ratio and Proportion, with this unit targeting 7.RP.2b, 7.RP.2c, and 7.RP.2d.

COHERENCE: As identified in the Table of Contents and further described in the Overview and the Foundational Standards, this unit is part of a larger unit that builds upon the development of understanding of proportional relationships (7.RP.2a) in topic A. In Topic A, the lessons build upon Grade 6 (6.RP.1, 6.RP.2, 6.RP.3) reasoning about ratios, rates and unit rates. Subsequent topics will have students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers and eventually applying the concepts in working with side lengths and areas of polygons. This demonstrates the connections within the domain, to other domains and with consideration to vertical alignment. The lessons in this module also demonstrate coherence as they flow through the progressions of understanding. The next module, Topic C, focuses on ratios and rates involving fractions (7.RP.1, 7.RP.3), which is the next step in the progression of ratio and proportion understanding.

Rigor: This unit meets the expectations for rigor in a variety of ways. There are ample opportunities for students to independently apply mathematical concepts in real-world situations as they solve challenging problems with a balance of application, conceptual understanding and procedural skill.

Application: Students have numerous opportunities to work independently and also in groups to solve problems that require perseverance and the opportunities to apply a variety of strategies.

Conceptual Understanding: The use of applications and problem solving lends itself to numerous opportunities to demonstrate a deep conceptual understanding as they work through the problems and engage in discussions with other students to make sense of the mathematics in relevant situations. An example is a lesson in the problem related to Al's Produce Stand and the closing questions related to it as students process the modeling of the proportional relationship in an equation. Students make connections and they model multiple representations of graphical and tabular

	<p>representations of proportional reasoning. The numerous examples of scaffolding are designed to help build the conceptual understanding and to extend understanding.</p> <p>Procedural Skill and Fluency: This set of lessons helps students gradually develop their procedural skills through numerous opportunities to identify and create the constant of proportionality in a variety of forms (tables, graphs and equations). The progressive connections and level of difficulty will facilitate student progress towards fluency .</p>
<p>Rating: 3 – Meets most to all of the criteria in the dimension</p>	

Dimension III – Instructional Supports

<p><i>The lesson/unit is responsive to varied student learning needs:</i></p> <ul style="list-style-type: none"> ✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media. ✓ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline. ✓ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking. ✓ Addresses instructional expectations and is easy to understand and use. ✓ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners. <ul style="list-style-type: none"> – Supports diverse cultural and linguistic backgrounds, interests and styles. – Provides extra supports for students working below grade level. – Provides extensions for students with high interest or working above grade level. <p><i>A unit or longer lesson should:</i></p> <ul style="list-style-type: none"> ✓ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share). ✓ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently. ✓ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ☐ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed 	<p>Clear and Sufficient Guidance: The lessons include clear and sufficient guidance to support teaching and learning of the targeted standards. The entire layout of the unit lends itself to ease of educator use. The Table of Contents gives the BIG Picture and the entire sequence of topics. The overview provides content focus, practices focus, terminology with definitions, suggested tools and representations and the assessment summary. The organization of Student Outcomes and Lesson Notes ensures that the educator knows the intent of the lesson. The Class work section details how to proceed with the lesson with directions on the strategies and intent of the activity. The inclusion of the embedded practices and questions to ask and to drive instruction and student progress towards the content and the practices are beneficial for educator understanding of how to proceed. There are examples of possible student responses to the problems. The Closing Question section provides strategies to bring the key elements of the lesson together and sets students up for the Exit Ticket, which is given the last five minutes of the lesson. The suggested time allotments are beneficial for pacing, but the actual time allocation might need to be more in response to the needs of the students.</p> <p>Productive struggle is evidenced through the numerous opportunities for problem solving activities that do not have immediate conclusions and that elicit mathematical thinking. These are typically at the launch of each lesson.</p> <p>There are examples of "telling students" things. Productive struggle might be better served if students draw those conclusions from their efforts. Students greatly benefit from the opportunity to truly struggle with the problems and make connections. Greater learning will prevail if students are given the time and and the positive environment that promotes productive struggle.</p> <p>Overall, the unit is easy to follow and understand because of the format and clear explanations and expectations. The use of the Problem Sets is found in the Lesson notes. These can vary, depending upon need.</p> <p>There are several examples of scaffolding for intervention and for extending thinking identified in scaffolding bubbles. These options can be quite beneficial for educators to use to meet the needs of diverse learners.</p> <p>This is a short unit within a larger unit. Within the small construct there is a gradual progression of learning(students move from the identification of the constant of proportionality to actually using it in equations and graphs) and within the larger unit, there appears to be continuity and appropriate</p>
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quickly and accurately.	progression for student learning.
Rating: 3 – Meets most to all of the criteria in the dimension	

Dimension IV – Assessment

<p><i>The lesson/unit regularly assesses whether students are mastering standards-based content and skills:</i></p> <ul style="list-style-type: none"> ✓ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS. ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts. ✓ Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance. <p><u><i>A unit or longer lesson should:</i></u></p> <ul style="list-style-type: none"> ✓ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures. 	<p>There are several levels of assessments embedded into the lessons (class activities, exit tickets and the Mid-Module Assessment). Since this unit is part of a larger unit, there is a mid-term assessment to apparently be used in a summative and possibly a formative manner. The intent of the results is not clearly identified. The items on the mid-term assessment are aligned with the expectations in the standards. The rubric for the assessment clearly helps determine the appropriate score points for the items representing the level of student understanding.</p> <p>It would be helpful to label the answer sheet and/or the student assessment sheet with the identified standards, making it easier to use for formative instructional practices. The End of Module assessment (unseen) seems to be a summative assessment of the larger unit.</p> <p>The lesson specific exit tickets are aligned with the expectations of the lesson and are expected to be completed by each student independently in the last five minutes of the class. The subsequent Lesson Notes indicate how the educator might use the results of the previous Exit Ticket in service of new student learning.</p> <p>The larger unit and this sub-unit might benefit from a form of pre-assessment to inform educators regarding the appropriate entry points for students.</p>
Rating: 3 – Meets most to all of the criteria in the dimension	

Summary Comments

<p>This is a quality unit which meets the key criteria in this rubric. This lesson is an exemplar that is clearly focused on a deep understanding of some of the most major work of grade 7.</p> <p>Some primary assets are:</p> <ul style="list-style-type: none"> - ease of use structure and purpose of each lesson and activity within the lesson, - allocating adequate time allowing students to evolve in their understanding, - clearly identifying a strategy to use the results of the ongoing assessments (class work and exit tickets) in formative instructional practices, - clearly embedding three of the Mathematical Practices, and - the quality and quantity of scaffolding for diverse learners.
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Rating Scales

Rating Scale for Dimensions I, II, III, IV:

3: Meets most to all of the criteria in the dimension

2: Meets many of the criteria in the dimension

1: Meets some of the criteria in the dimension

0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:

E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (**total 11 – 12**)

E/I: Exemplar *if* Improved – Aligned and needs some improvement in one or more dimensions (**total 8 – 10**)

R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (**total 3 – 7**)

N: Not Ready to Review – Not aligned and does not meet criteria (**total 0 – 2**)

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:

- 3: Exemplifies CCSS Quality** - meets the standard described by criteria in the dimension, as explained in criterion-based observations.
- 2: Approaching CCSS Quality** - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.
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- 1: Developing toward CCSS Quality** - needs significant revision, as suggested in criterion-based observations.
- 0: Not representing CCSS Quality** - does not address the criteria in the dimension.

Descriptor for Overall Ratings:

- E: Exemplifies CCSS Quality** – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.
- E/I: Approaching CCSS Quality** – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.
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- R: Developing toward CCSS Quality** – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.
- N: Not representing CCSS Quality** – Not aligned and does not address criteria.